

# Joe Holbrook Memorial Math Competition

5th Grade

October 22, 2023

## General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

## Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is the greatest two-digit number that, when tripled, is still a two-digit number?
2. Evaluate  $\frac{111}{1+1+1}$ .
3. Snorlax subtracts 10 from the number 2023 until a one-digit number remains. What is that one-digit number?
4. James does twenty push-ups a day every day for two weeks. How many push-ups does he do in total?
5. How many inches longer is 7.7 feet than 2.2 feet?
6. Help! My pet ducks ran away, and I can't find them! They ran 50 meters in some direction, so I ran 27 meters in a random direction hoping to catch them. What is the greatest possible distance (in meters) between me and my ducks right now?
7. If I were to split 47 cookies evenly between me and my 5 friends so we each get a whole number of cookies, and keep the remaining cookies to myself, how many cookies would I have in the end?
8. Alice is trying to guess Bob's secret number. She knows it is an odd prime number that is one more than triple a square number. What is the smallest possible value for Bob's secret number if it is not 13? A square number,  $n$ , is a number such that  $n = p \times p$  for some number  $p$ .
9. Every day, the height of Mr. Tree increases by the same amount. Yesterday, Mr. Tree was 87 feet tall, and today, Mr. Tree is 100 feet tall. How tall, in feet, will Mr. Tree be tomorrow?
10. Snivy planned to add 3 grams of salt and 30 grams of sugar to a bowl, but he instead added 30 grams of salt and 3 grams of sugar to the bowl. Not wanting to waste ingredients, how much more sugar should he add to the bowl to match the intended ratio of salt to sugar?
11. From the list of integers 1, 2, 3, 4, 5, 6, 7, 8, which number should be removed so that the sum of the remaining integers is a multiple of 11?
12. What is the product of all the single-digit prime numbers?
13. Arnav has to fly to Hawaii for a chess tournament! He has to fly in 3 planes, and the companies he has available are United, American, and Southwest. He can only fly each company once. How many orders of flights can he take if he can only take each company once, if he refuses to fly United first?
14. A square of side length 420 is divided into 49 smaller squares each with the same side length. What is the side length of one of the 49 squares?
15. Carl is deciding what he wants to wear to school. He has 3 shirts, 4 pairs of pants, and 2 hats to choose from. If he has to wear 1 shirt and 1 pair of pants but can choose whether or not to wear a hat, how many combinations of clothes can he wear?
16. We define the notation  $(a\clubsuit b)$ , for positive integers  $a$  and  $b$  to be the difference between the value of the least common multiple of  $a$  and  $b$  and the value of the greatest common divisor of  $a$  and  $b$ . If  $a$  and  $b$  are chosen to be divisors of 36, what is the maximum value of  $(a\clubsuit b)$ ?
17. Bob has 10 matching pairs of mittens in a basket, each of a different color. He has just woken up and randomly picks mittens out of the basket. How many does he have to pick to guarantee that he has a matching pair of mittens?
18. Out of a group of birds, exactly  $\frac{1}{3}$  can fly and the rest cannot. If 5 of the birds that cannot fly are penguins, what is the minimum possible number of birds in the group?
19.  $45^2 = 2025$  is the next year that is a perfect square. How many years will it take after 2025 for it to be a perfect square year again?
20. Christin and Joy bake cookies every day. On the first day, Christin and Joy bake 10 and 0 cookies, respectively. With each successive day, Christin bakes 2 more cookies than she did the day before and Joy bakes 4 more than she did the day before. After how many days will Joy and Christin have baked the same total amount of cookies?
21. There are three positive integers written on three pieces of paper, and one of the integers is the sum of the other two. Knowing this, Shining looks at the integers on the first two pieces of paper, and remarks, "The integer on the third piece of paper must be 46!" What is the number on the first piece of paper?

22. A restaurant offers to sell a 16-inch diameter pizza for half the price of a 20-inch diameter pizza! The 16-inch pizza provides  $k\%$  more square inches of pizza per dollar than the 20-inch pizza. Find  $k$ .
23. Let  $P$  be a point inside the rhombus  $ABCD$  such that the distance from  $P$  to  $AB$ ,  $BC$  and  $CD$  respectively are 20, 23, 2023 respectively. Find the distance from  $P$  to  $AD$ .
24. Seedot folds a  $6 \times 6$  piece of origami paper along its diagonal. He then rips a  $2 \times 2$  square out of the right-angled corner of the resulting triangle. What is the area of the (now ripped) origami paper when unfolded?
25. What is the sum of the digits of  $10^{2023} - 10^{2006}$ ?
26. In a class of 100 students, 30% raise their left hand, 25% raise their right hand, and 20% raise both hands. How many students do not raise either hand?
27. The Quagmires are making a pancake tower. This tower is made of 3 pancakes with radii 6 inches, 4 inches, and 2 inches stacked, centered, and in that order. Each pancake is a cylinder with a height of 1 inch. If the Quagmires cover the pancake in syrup (not including the bottom of the pancakes), the Quagmires need  $a\pi$  square inches of syrup. What is  $a$ ?
28. In a square with side length 1, a line is drawn through the center of the square. The reflection of the line over a diagonal of the square is also drawn, dividing the square into four regions. Given that the ratio of the area of the smallest region to the largest region is  $20 : 23$ , then the area of the smallest region can be expressed as  $\frac{p}{q}$ , in simplest terms. Find  $p + q$ .
29. Piplup created a  $4 \times 4 \times 4$  wooden cube by combining sixty-four  $1 \times 1 \times 1$  wooden cubes. Between any two wooden cubes sharing a face, Piplup used 1 drop of glue. How many drops of glue did Piplup use in total?
30. Esme the octopus labels her eight legs with consecutive numbers starting from 1. She writes a 1 on her first leg, a 2 on her second leg, a 3 on her third leg, and so on. When she reaches the number 9, she writes that on her first leg and she writes the number 10 on her second leg, and so on. She writes up to the number 35. If the number she is writing is composite, she writes it in blue, and if the number she is writing is not composite she writes it in red. How many of her legs have numbers of all the same color on it?
31. Alice knows how to correctly use PEMDAS, but Bob mistakenly does the operations in reverse! In other words, he does addition and subtraction before multiplication and division. They are both trying to evaluate
- $$1 \_ 2 \_ 3 \_ 4,$$
- where the blank spaces can be replaced by any of  $+$ ,  $-$ ,  $\times$  or  $\div$ . What is the maximum possible difference between their answers for their individual choices of operations?
32. In some numbers, the number of times a digit shows up in the number is the value of the digit. For example, in 5255525, 2 shows up 2 times and 5 shows up 5 times. How many of these numbers exist if they must be 7 digits long?
33. Consider the square of integer coordinate points with vertices of  $(0, 0)$ ,  $(0, 4)$ ,  $(4, 4)$  and  $(4, 0)$ . How many isosceles triangles with all vertices on integer coordinate point can be formed with a side on the  $x$  axis?
34. In a 20-person ping-pong tournament, a player is eliminated after losing their fifth game, and games are played until one person remains. What is the maximum possible number of games played?
35. The graph of  $y = |ax^2 - b|$  intersects the line  $y = 4$  at exactly 3 points. If the the longest distance between two of these three intersections is 4 and  $a > 0$ , what is  $a$ ?
36. Masha the mongoose lives on a path around the edge of a circular park with a radius of 6 kilometers. Starting at home, Masha walks the path for  $8\pi$  kilometers, then decides to cut through the park to walk back home in a straight line. If Masha had to walk  $a\sqrt{b}$  kilometers to get home ( $a$  and  $b$  are positive integers and  $b$  is not divisible by the square of any prime number), find  $a + b$ .
37. The process of obtaining the "trisection" of a regular  $2n$ -gon is as follows: trisect each of the  $2n$  segments connecting a vertex to the center, alternate between selecting the trisected point closer to the center and further away from the center, connect these points to form a polygon. What is the area of the trisection of a regular hexagon with area 36?

38. A *perplexing* number is a square number with the same amount of digits as the number in the units digit. For example, 1 would be a perplexing number as it is a square number, has 1 digit, and the number in the units digit is 1. Find the sum of the first 3 perplexing numbers.
39. How many subsets of positive integers from 1 – 17 have the least common multiple of all the numbers in the subset as its largest element?
40. When the number  $555\dots555_6$  with 2023 occurrences of the digit 5 is converted to base 36, the sum of the digits of the resulting base 36 number is  $S$ . Find  $S$ .