Joe Holbrook Memorial Math Competition

7th Grade

October 22, 2023

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. A volunteer fire department responds to an average of 3 calls each day. On average, how many calls do they respond to in a non-leap year?

2. Compute $(1 + 2 \times 3) - (1 \times 2 + 3)$.

3. In the Reptile Room, for every turtle, there are two lizards and for every lizard, there are 3 snakes. If there are 20 turtles, how many snakes are there?

4. Alice is trying to guess Bob's secret number. She knows it is an odd prime that is one more than triple a square number. What is the smallest possible value for Bob's secret number if it is not 13?

5. Compute $2021 \cdot 2022 - 2023 \cdot 2024$.

6. Carl is deciding what he wants to wear to school. He has 3 shirts, 4 pairs of pants, and 2 hats to choose from. If he has to wear 1 shirt and 1 pair of pants but can choose whether or not to wear a hat, how many combinations of clothes can he wear?

7. Bob has 10 matching pairs of mittens in a basket, each of a different color. He has just woken up and randomly picks mittens out of the basket. How many does he have to pick to guarantee that he has a matching pair of mittens?

8. When I add 10 to my favorite number, triple the result, take the square root of that result, and add 8, I get my favorite number again. What is my favorite number?

9. The wheels on the bus go round and round at a rate of 14 full rotations per second. If the wheels have a radius of $\frac{3}{\pi}$ feet, how far, in feet, does the bus travel in one minute?

10. In triangle $\triangle ABC$, $\angle ABC = \angle ACB$. If two of the triangle's sides are 7 and 5, what is the sum of the possible lengths of the third side?

11. Ellie wrote a number divisible by 9 on a whiteboard, but Elliott erased the last digit! If the whiteboard currently says 8273, what digit did Elliott erase?

12. Seedot folds a 6×6 piece of origami paper along its diagonal. He then rips a 2×2 square out of the right-angled corner of the resulting triangle. What is the area of the (now ripped) origami paper when unfolded?

13. Suppose $pq + qr = 243$ for certain primes p, q, r. Find pqr.

14. The Quagmires are making a pancake tower. This tower is made of 3 pancakes with radii 6 inches, 4 inches, and 2 inches stacked, centered, and in that order. Each pancake is a cylinder with a height of 1 inch. If the Quagmires cover the pancake in syrup (not including the bottom of the pancakes), the Quagmires need $a\pi$ square inches of syrup. What is a?

15. Anthony has a perfectly spherical balloon he wants to fit into his perfectly cubical box. The volume of the box is 216 ft^3. If he makes the biggest spherical balloon that could fit in the box, the difference between the volume of the box and the balloon can be expressed as $a - b\pi$, where a and b are positive integers. Find $a + b$.

16. What is the units digit of $7^{2023} + 9^{2023}$?

17. Masha the mongoose lives on a path around the edge of a circular park with a radius of 6 kilometers. Starting at home, Masha walks the path for 8π kilometers, then decides to cut through the park to walk back home in a straight line. If Masha had to walk $a\sqrt{b}$ kilometers to get home, where a and b are positive integers and b is not divisible by the square of any prime number, find $a + b$.

18. Elena and Lila are playing a game where they each roll one fair, 6-sided die. Lila wants her roll to be at least 2 greater than Elena's. The probability of this occurring can be expressed as a fraction in simplest form $\frac{m}{n}$. What is mn?

19. Two circles are centered around the origin, one with radius 2 and the other with radius 14. What is the ratio of the area in the larger circle but not in the smaller to the area of the smaller circle?

20. Let x, y, $x + y$, xy form an arithmetic sequence in that order for some nonzero real numbers x and y. Compute the value of y.

21. For some digits A and B, the equation $\underline{AB} + \underline{B9} = \underline{16A}$ is true. What is the product of A and B? (Note: $\underline{AB}$ denotes the number with the two digits A and B in that order, not $A \cdot B$)

22. An equiangular hexagon has 3 sides of length 4 and 3 sides of length 2. If the area of the hexagon can be expressed as $a\sqrt{b}$ where a and b are positive integers and b is not divisible by the square of any prime number, find $a + b$.

23. How many distinct arrangements are there of the letters BCAMCJHMMC? For example, BCAMCJHMCM and BCAMCJHMMC count as one each.

24. Triangle $\triangle ABC$ is inscribed in a circle centered at O with radius 2. $\overline{BC}$ has length $3\frac{1}{5}$ and is perpendicular to ray $\overrightarrow{\mathrm{AO}}$. The area of $\triangle ABC$ can be represented as the mixed number $x\frac{y}{z}$ for positive integers x, y, and z, with $y < z$ and y relatively prime to z. Find $x + y + z$.

25. Esme the octopus labels her eight legs with consecutive numbers starting from 1. She writes a 1 on her first leg, a 2 on her second leg, a 3 on her third leg, and so on. When she reaches the number 9, she writes that on her first leg and she writes the number 10 on her second leg, and so on. She writes up to the number 35. If the number she is writing is composite, she writes it in blue, and if the number she is writing is not composite she writes it in red. How many of her legs have numbers of all the same color on it?

26. How many more nondegenerate triangles can be formed from the vertices of a nonagon (a 9-sided polygon) compared to an octagon (an 8-sided polygon)?

27. A regular tetrahedron is sliced parallel to one of its bases (keeping the other corner), so that the surface area remaining is $\frac{4}{9}$ of the original. If the fraction of the height that was cut off can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?

28. Two circles of radius $\frac{12}{\pi}$ are overlapped such that the center of each circle is on the other circle's boundary. What is the perimeter of the region of points covered by at least one of the two circles?

29. Kelvin the Frog is on an 8×8 chessboard with sides parallel to the cardinal directions (north/south, east/west), starting on the southwesternmost square. Every second, he moves to a square that is diagonally adjacent to the one he is currently on (for example, his first move must be one step northeast). How many ways can he get to the northeasternmost square in exactly nine seconds?

30. A square of unknown side length has one vertex located at $(0, 0)$ and another vertex located at $(6, 8)$. Find the positive difference between the maximum and minimum possible area of the square.

31. Daniel, Andrea, Jeremy, and Anthony have a group chat. Each person sent either 10, 30, 56, or 77 messages, but none of them sent the same number of messages as anyone else. Of the following statements, exactly ONE statement is false.

    (a) Anthony sent a total of > 40 messages.
    (b) The GCD of the number of messages sent by Andrea and Jeremy is 10.
    (c) The number of divisors of the number of messages Daniel sent is 8.
    (d) The number of messages Daniel sent is not a multiple of 8 and is not a multiple of 5.
    (e) The GCD of the number of messages sent by Anthony and Jeremy is 1.

    How many messages did Daniel send?

32. Jenny is an efficient pizza worker. What is the largest number of pieces she can cut out of a pizza with exactly 6 cuts?

33. Mark has a calculator with two buttons, one which multiplies by 3 and adds 2, and one which multiplies by 9 and adds 8. The calculator currently says 1, and will produce an error if the number reaches over 2023. How many ways are there for Mark to use the buttons to make the calculator error?

34. A sphere of radius 1 is inscribed in a cube, and a line segment connects one point where a face of the cube is tangent to the sphere to a vertex on the opposite side of the cube. If $\frac{a}{b}$ of the segment's total length lies outside the sphere, where a and b are relatively prime positive integers, compute $a + b$.

35. Find the value of $(x + 2)^3$ if $x^4 + 8x^3 + 24x^2 + 29x + 10 = 0$, where $x \neq -2$.

36. Alice and Bob have a secret code that could potentially end the world! Luckily, we managed to uncover their secret messages, which were encrypted by replacing every digit with its own respective letter.

    - Bob: What was the code again? I know it is either BEE, EBE, or EEB, but I forgot which one.
    - Alice: If you remembered the code was divisible by E, you could've realized it was BEE without asking!
    - Bob: Sorry, I only remembered that the product of the digits of the code was SET, which didn't help...

    What is the secret code?

37. Let $\triangle ABC$ satisfy $AB = 20$, $BC = 22$ and $\angle ABC$ is obtuse. Point D is on the line parallel to AB passing through C such that BD bisects $\angle ABC$. Finally, let E be the point on AD such that CE bisects $\angle BCD$. Given that $AE = 1$, compute the length of DE.

38. What is the sum of all integers x such that $2xy + 7x - 8y = 0$, for some integer y?

39. We start with a string of n ones, and insert some positive number of plus signs between them to create an expression. For example, if we start with 3 ones, we can form $11 + 1$, $1 + 1 + 1$, or $1 + 11$, evaluating to 12, 3, and 12, respectively. For how many positive integer values of n can we create an expression that evaluates to 123?

40. Let p and q be odd primes with $p^2 + pq + q = 3985$. Find $p + q$. (The original problem stated positive primes, which allowed two different solutions. Both solutions were counted as correct.)