Answer Key

Name: GRADE 4 KEY **ID**: GRADE 4 KEY

1 . 13	11 . 2023	21 . 19	31 . 59
2 . 37	12 . 1260	22 . 28	32 . 48
3 . 11	13 . 109	23 . 256	33 . 0
4 . 66	14 . 67	24 . 9	34 . 22
5 . 113	15 . 60	25 . 1978	35 . 0
6 . 12	16 . 17	26 . 53	36 . 120
7 . 60	17 . 36	27 . 17	37 . 144
8 . 4	18 . 101	28 . 65	38 . 288
9 . 210	19 . 9	29 . 12022	39 . 36
10 . 11	20 . 31	30 . 153	40 . 169

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4

Total Score:

Joe Holbrook Memorial Math Competition

4th Grade

October 22, 2023

1. 50 packing peanuts - 37 packing peanuts = $\boxed{13}$ packing peanuts left.
2. $\dfrac{111}{1+1+1} = \dfrac{111}{3} = \boxed{37}$.
3. $73 \div 7 = 10$ remainder 3, so I have completely read 10 full chapters and am currently reading chapter $\boxed{11}$.
4. The difference between 7.7 feet and 2.2 feet is 5.5 feet which is $5.5 \times 12 = \boxed{66}$ inches.
5. Mr.Tree's height increases by $100-87 = 13$ feet every day. Therefore, tomorrow Mr.Tree will be $100+13 = \boxed{113}$ feet tall.
6. First, 47 cookies are split among 6 people. Since $47 \div 6 = 7$ remainder 5, this means each person receives 7 cookies, and 5 cookies remain. These 5 cookies are added to the 7 cookies I already have, so I end with a total of $7 + 5 = \boxed{12}$ cookies.
7. 30% of 200 is $200 \times \dfrac{30}{100} = \boxed{60}$ poisoned apples.
8. To use the least number of coins, we should use as many large-valued coins as possible. Thus, to make 37 cents, we would use 1 quarter, 1 dime, and 2 pennies, for a total of $\boxed{4}$ coins.
9. The one-digit prime numbers are 2, 3, 5, and 7. Their product is $\boxed{210}$.
10. Consider the worst-case scenario. In this scenario, Bob selects 10 different mittens(one of each color), but if he picks one more, then he must have a mitten that matches with another. Thus, Bob must pick $\boxed{11}$ mittens.
11. We can compute directly to find that the expression equals $17^2 \times 7 = \boxed{2023}$.
12. Let c denote the number of cups that Sceptile sells per day *before* the price change. *Before* the price change, she makes $60c$ cents, whereas *after* the price change, she makes $70(c-3)$ cents. Since these two quantities are equal, $60c = 70(c-3)$. Solving for c gives $c = 21$, so Sceptile's makes $60 \times 21 = \boxed{1260}$ cents each day.
13. Since Bob's number is odd, all odd squares can be disregarded, as 3 times an odd square + 1 is always even. Consider the first several even squares: 4, 16, 36. The numbers generated by multiplying them by three and adding 1 are 13, 49, 109. Since it is known that 13 is not Bob's number, and 49 isn't prime, the first odd prime would be $\boxed{109}$.
14. By trading all his cookies for carrots, Elmo can get a maximum of $A = 10 + 30 \times 3 = 100$ carrots. By trading all his carrots for cookies, he can get a maximum of $30 + 10 \div 3 = 33\frac{1}{3}$ cookies, implying $B = 33$. Then $A - B = 100 - 33 = \boxed{67}$.
15. The area of the original square is 420×420, so each of the smaller squares must have an area of $\dfrac{420 \times 420}{49} = \dfrac{420 \times 420}{7 \times 7} = 60 \times 60$. Thus, each of the smaller squares must have a side length of $\boxed{60}$.
16. We can write Alyssa's age x years from now as $31 + x$ and James' age as $7 + x$. To find when Alyssa is twice as old as James, we can solve for x when $31 + x = 2 \times (7 + x)$. Solving this gives $x = 17$, meaning Alyssa will be twice as old as James in $\boxed{17}$ years.
17. Carl has 3 options for shirts, 4 options for pants, and 3 options for hats (since he can wear one of the 2 hats or wear no hat). Therefore, he has $3 \times 4 \times 3 = \boxed{36}$ combinations of clothes he can wear.

18. Notice that such an x would satisfy $x = \frac{x \times x}{101}$. Multiplying both sides by 101 and dividing both sides by x (which we can do since $x \neq 0$), we get $x = \boxed{101}$.

19. Since $\frac{1}{3}$ of the birds can fly, the total number of birds must be a multiple of 3. $\frac{2}{3}$ of the number of birds must also be greater than or equal to 5 implying the total number of birds must be greater than or equal to $\frac{15}{2}$. The smallest number that satisfies both conditions is $\boxed{9}$.

20. Let the number of children who bought tickets be c and the number of adults who bought tickets be a. Since the Caligari Carnival made 453 dollars from ticket sales, we know $7a+4c = 453$, and since 78 people bought tickets we know $a + c = 78$. Multiplying this second equation by 4 and subtracting it from the first equation, we have $3a = 141 \rightarrow a = 47$. Plugging in a in the first equation, we can then find that $c = 31$. Therefore, $\boxed{31}$ children bought tickets.

21. Since the number has a remainder of 4 when divided by 5, it has to end in a 4 or a 9. Since the number has a remainder of 3 when divided by 4, the number must be odd and therefore must end in a 9. Since we know the number ends with a 9, and we also know that the number is one less than a multiple of 4, we can just find the smallest multiple of 4 that ends with a 0 and subtract 1. 20 is the smallest multiple of 4 that ends with a 0, so Devin's favorite number is $20 - 1 = \boxed{19}$.

22. The area of the original 6×6 square of origami paper is $6 \times 6 = 36$ square units. The ripped origami paper has two 2×2 squares removed from its opposite corners. Each of the two 2×2 squares accounts for 4 square units of removed paper, for a total of 8 square units removed. So the area is now $36 - 8 = \boxed{28}$ square units.

23. We have four choices for each digit in a four-digit number, meaning that there are $4 \times 4 \times 4 \times 4$ powerful four-digit numbers. Thus, there are $\boxed{256}$ powerful four-digit numbers.

24. If Jack's square has a side length of 3 cm, the area of his square will be $3 \times 3 = 9$ cm^2. Caleb's square will have 9 times the area, so his square will have an area of $9 \times 9 = 81$ cm^2. Since $\sqrt{81} = 9$, Caleb's square will have a side length of $\boxed{9}$ cm.

25. The only numbers we have to exclude are the perfect squares. Since $45^2 = 2025$, there are 44 perfect squares less than 2023, and since there are 2022 numbers, our final answer is $2022 - 44 = \boxed{1978}$.

26. Notice that the square consists of two pairs of two congruent regions. Therefore, the combined area of the smallest and largest regions is equal to half the area of the square, which is $\frac{1}{2}$. Since the smallest region makes up $\frac{20}{20+23} = \frac{20}{43}$ of the total area of the smallest and largest regions, the smallest region has area $\frac{1}{2} \times \frac{20}{43} = \frac{10}{43}$. Therefore, the desired answer is $10 + 43 = \boxed{53}$.

27. Start by factorizing $2023 = 7 \times 17 \times 17$. We need one three-digit number and one two-digit number that multiply to 2023. Notice that this can be accomplished by multiplying $17 \times 7 = 119$ and 17. These two numbers fit our equation, so $E + L + D = 1 + 9 + 7 = \boxed{17}$.

28. A total of $30\% - 20\% = 10\%$ of students, or 10 students, raise only their left hand, and similarly $25\% - 20\% = 5\%$ of students, or 5 students, raise only their right hand. And 20% of students, or 20 students, raise both hands. So $5 + 10 + 20 = 35$ students have at least one hand raised, leaving $100 - 35 = \boxed{65}$ students without any hands raised.

29. Let $x = 0.\overline{2023}$. Then, $10000x = 2023.\overline{2023}$. $10000x - x = 9999x = 2023.\overline{2023} - 0.\overline{2023} = 2023$. So, $9999x = 2023$, $x = \frac{2023}{9999}$. Thus, $p = 2023$. $q = 9999$, and $p + q = \boxed{12022}$.

30. This problem is easiest when you factor 10^{2006} out (as it will only add zeroes on the end). $10^17 - 1$ will have 17 9's, meaning the answer is $17 \times 9 = \boxed{153}$.

31. For Alice, notice that the largest value she can get is $1 + 2 \times 3 \times 4$. This is because multiplication leads us to the largest value, and multiplying by 1 does not increase the value, so we change that to +. This gives us a maximum of 25. Alice's least value is $1 - 2 \times 3 \times 4 = -23$. On the other hand, Bob's largest value can be found to be $1 + 2 \times 3 \times 4 = (1 + 2) \times 3 \times 4 = 36$. This is because we want to utilize the fact that Bob does addition first. Then, his smallest value is $1 - 2 - 3 \times 4 = -16$, because Bob needs to have a large negative which he can achieve by doing subtraction first. So, we either have that the difference is $|25 - (-16)| = 41$, or $|36 - (-23)| = 59$. So the answer is $\boxed{59}$.

32. The area of the smaller circle is 4π, while the area of the larger circle is 196π. Subtracting these areas will give the area outside of the smaller circle but still inside the larger circle, which is 196π - $4\pi = 192\pi$. The ratio of this area to the area of the smaller circle, is $\frac{192\pi}{4\pi} = \boxed{48}$.

33. Notice that $10002 = 2 \times 5001$. Because 2 and 5001 are both factors of N. and 5001 is not divisible by 2, this also means that $2 \times 5001 = 10002$ is a factor of N. So, the remainder when N is divided by 10002 is $\boxed{0}$.

34. We can work recursively. Let p_n be the maximum number of pieces that can be created with n cuts. Notice that to maximize the number of pieces created after the $n + 1$th cut, the $n + 1$th line must cross all other n lines inside the circle, but not pass through any pairwise intersections between lines.

    As the $n + 1$th line will pass through each of the n line segments, it gets split into $n + 1$ segments. Each of these segments splits a piece that was already formed by n cuts in two, thus adding $n + 1$ pieces.

    So, the base case must be with $n = 1$, which can only split the pizza into two slices. Thus, $p_2 = p_1 + 2 = 3$. Continuing, we obtain that $p_6 = 2 + 2 + 3 + 4 + 5 + 6 = \boxed{22}$.

35. We write out the first 15 terms. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610. We note that the last number is even. Each even number is the sum of the previous 2 odd numbers, so when calculating $x - y$, all even terms cancel out with the previous 2 odd terms (for example, $2 - 1 - 1 = 0$ and $8 - 5 - 3 = 0$). So $x - y = \boxed{0}$. (Note that the observation that the last term is even was sufficient; we didn't have to calculate all 15 terms).

36. The side length is $\frac{52}{4} = 13$ and the half-diagonal length is 5. Therefore, the other half-diagonal length is $\sqrt{13^2 - 5^2} = 12$. The area is $12 \times 5 \times 2 = \boxed{120}$.

37. Observe that every face of every $1 \times 1 \times 1$ cube is in one of two positions: touching a drop of glue, or facing the outside of the $4 \times 4 \times 4$ cube. There are $6 \times 4^2 = 96$ faces on the outside of the $4 \times 4 \times 4$ cube and $6 \times 4^3 = 384$ faces in total, so $384 - 96 = 288$ faces are touching a drop of glue. Since every drop of glue can be associated with 2 faces touching a drop of glue, there must be $288 \div 2 = \boxed{144}$ drops of glue.

38. Suppose the pair is (a, b). There are 16 options for their tens digits: $(1, 2)$, $(2, 3)$, through $(8, 9)$, or $(2, 1)$, $(3, 2)$, through $(9, 8)$. For their unit digits, $(0, 1)$ and $(1, 0)$ are also options, so there are 18 options in total. For example, if $(7, 6)$ and $(0, 1)$ are chosen, our pair would be $(70, 61)$. So, the answer is $16 \times 18 = \boxed{288}$.

39. We proceed by casework based on the type of isosceles triangle. Suppose one of the equal sides of the triangle has its base on the x-axis. Now, we can do casework depending on what the side length of this triangle is. If it is 1, there are 8 of such triangles, and if it is 2, there are 6, and if it is 3, there are 4 of such triangles, and if the side length is 4, there are 2. Thus, there are $8 + 6 + 4 + 2 = 20$ of such triangles. Our second case is if the base of the isosceles triangle falls on the x-axis. Therefore, the length of the base must be even, in order to have the third vertex be on a lattice point with a x coordinate directly centered. So, we begin having a base of length 2. There are 3 such segments, and each have 4 triangles corresponding to the segment, making 12 triangles total. If the length of the base is 4, there is only one such base, and 4 triangles. So, there are $12 + 4 = 16$ triangles in this case. In total, we have $\boxed{36}$ isosceles triangles.

40. We figure out what digits work, and then calculate how many ways there are to rearrange those digits. We note that in the example given, 5255525, there are two 2's and five 5's. So considering other possibilities, there are seven 7's, one 1 and six 6's, two 2's and five 5's, three 3's and four 4's, and finally, one 1, one 2, and one 4. For each case, we can reorder the digits as needed.
    For 7777777, there's 1 ordering.
    For 1666666, there's $\binom{7}{1} = 7$ orderings.
    For 2255555, there's $\binom{7}{2} = 21$ orderings.
    For 3334444, there's $\binom{7}{3} = 35$ orderings.
    For 1224444, there's $\frac{7!}{4!2!} = 105$ orderings.
    In total, there are $1 + 7 + 21 + 35 + 105 = \boxed{169}$ numbers.