Answer Key

Name: GRADE 5 KEY **ID**: GRADE 5 KEY

1 . 33	11 . 3	21 . 23	31 . 59
2 . 37	12 . 210	22 . 28	32 . 169
3 . 3	13 . 4	23 . 2020	33 . 36
4 . 280	14 . 60	24 . 28	34 . 99
5 . 66	15 . 36	25 . 153	35 . 2
6 . 77	16 . 35	26 . 65	36 . 9
7 . 12	17 . 11	27 . 60	37 . 8
8 . 109	18 . 9	28 . 53	38 . 2469
9 . 113	19 . 91	29 . 144	39 . 111
10 . 297	20 . 11	30 . 3	40 . 35390

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4

Total Score:

Joe Holbrook Memorial Math Competition

5th Grade

October 22, 2023

1. Since $3 \times 33 = 99$ but $3 \times 34 = 102$, the answer is $\boxed{33}$.
2. $\dfrac{111}{1+1+1} = \dfrac{111}{3} = \boxed{37}$.
3. The one-digit number should be (by definition!) the remainder when 2023 is divided by 10, or $\boxed{3}$ since the last digit of 2023 is a 3.
4. There are 7 days per week, so 2 weeks consist of 14 days. Each day, James does 20 pushups, so we can multiply 14×20 to get a total of $\boxed{280}$ pushups.
5. The difference between 7.7 feet and 2.2 feet is 5.5 feet which is $5.5 \times 12 = \boxed{66}$ inches.
6. The worst possible situation is that I ran in the opposite direction as my ducks, in which case the distance between us would be $50 + 27 = \boxed{77}$ meters. :(
7. First, 47 cookies are split among 6 people. Since $47 \div 6 = 7$ remainder 5, this means each person receives 7 cookies, and 5 cookies remain. These 5 cookies are added to the 7 cookies I already have, so I end with a total of $7 + 5 = \boxed{12}$ cookies.
8. Since Bob's number is odd, all odd squares can be disregarded, as 3 times an odd square + 1 is always even. Consider the first several even squares: 4, 16, 36. The numbers generated by them are 13, 49, 109. Since it is known that 13 is not Bob's number, and 49 isn't prime, the first such odd prime would be $\boxed{109}$.
9. Mr. Tree's height increases by $100 - 87 = 13$ feet every day. Therefore, tomorrow Mr. Tree will be $100 + 13 = \boxed{113}$ feet tall.
10. The intended ratio of salt to sugar is $1 : 10$, so since there are already 30 grams of salt, there should be 300 grams of sugar in the end, meaning $300 - 3 = \boxed{297}$ grams of sugar should be added.
11. Notice that the sum of the 8 integers is 36. Since 36 leaves a remainder of 3 when divided by 11, we must remove a number that leaves a remainder of 3 when divided by 11. The only number that satisfies this on the list is 3, so we must remove the number $\boxed{3}$.
12. The one digit primes are 2, 3, 5, and 7, so $2 \times 3 \times 5 \times 7 = \boxed{210}$.
13. Since Arnav can't fly United first, he has 2 options for his first flight. For his second flight, he can choose from either of the 2 remaining options, and after that his third flight must be by the only option that is remaining. $2 \times 2 \times 1 = \boxed{4}$ different orders of flights he could take.
14. The area of the original square is 420×420, so each of the smaller squares must have an area of $\dfrac{420 \times 420}{49} = \dfrac{420 \times 420}{7 \times 7} = 60 \times 60$. Thus, each of the smaller squares must have a side length of $\boxed{60}$.
15. Carl has 3 options for shirts, 4 options for pants, and 3 options for hats (since he can wear one of the 2 hats or wear no hat). Therefore, he has $3 \times 4 \times 3 = \boxed{36}$ combinations of clothes he can wear.
16. We can observe that the maximum value of the least common multiple of two divisors of 36 is 36 itself. Similarly, the minimum value of the greatest common divisor is 1, which occurs when a and b are relatively prime. This is possible when $a = 4$ and $b = 9$. Therefore, our answer is $36 - 1 = \boxed{35}$.
17. Consider the worst-case scenario. In this scenario, Bob selects 10 different mittens(one of each color), but if he picks one more, then he must have a mitten that matches with another. Thus, Bob must pick $\boxed{11}$ mittens.

18. Since $\frac{1}{3}$ of the birds can fly, the total number of birds must be a multiple of 3. $\frac{2}{3}$ of the number of birds must also be greater than or equal to 5 implying the total number of birds must be greater than or equal to $\frac{15}{2}$. The smallest number that satisfies both conditions is $\boxed{9}$.

19. This problem is solved quickly with using the difference of squares formula. We want to find the difference $46^2 - 45^2$, which is factored into

$$(46 + 45)(46 - 45) = (91)(1) = \boxed{91}$$

20. After $n + 1$ days, Christin will have baked a total of $10 + (10 + 2) + (10 + 2 \times 2) + \ldots + (10 + 2 \times n) = 10(n + 1) + 2 \times \frac{n(n + 1)}{2} = (10 + n)(n + 1)$ cookies. Joy will have baked a total of $0 + 4 + 8 + \ldots \times 4n = 4 \times \frac{n(n + 1)}{2} = 2n(n + 1)$. They will have baked the same amount when $(10 + n)(n + 1) = 2n(n + 1)$ or $10 + n = 2n \implies n = 10$, suggesting that they had baked the same number of cookies after $n + 1 = 10 + 1 = \boxed{11}$ days.

21. Notice that if he saw two distinct integers, he could not have concluded this, since both the absolute difference and sum of these two integers could have been on the third piece of paper. Therefore, he must have seen the same two integers. In addition, since the difference when an integer is subtracted from itself is 0, 46 must be the sum of the two integers. Therefore, the integer on the first piece of paper is $\frac{46}{2} = \boxed{23}$.

22. The 16-inch pizza has $\frac{16}{20} \times \frac{16}{20} = \frac{16}{25}$ times the area as the 20-inch pizza, so it has $\frac{16}{25} \times 2 = \frac{32}{25}$ times more square inches of pizza per dollar than the 20-inch pizza, which is a $\boxed{28}$ percent increase.

23. Notice that the distances from P to AB and CD must have the same sum as the distances from P to BC and AD due to the area formula for a parallelogram. Therefore, the desired distance is $2023 + 20 - 23 = \boxed{2020}$.

24. The area of the original 6×6 square of origami paper is $6 \times 6 = 36$ square units. The ripped origami paper has two 2×2 squares removed from its opposite corners. Each of the two 2×2 squares accounts for 4 square units of removed paper, for a total of 8 square units removed. So the area is now $36 - 8 = \boxed{28}$ square units.

25. This problem is easiest when you factor 10^{2006} out (as it will only add zeroes on the end). $10^{17} - 1$ will have 17 9's, meaning the answer is $17 \times 9 = \boxed{153}$

26. A total of $30\% - 20\% = 10\%$ of students, or 10 students, raise only their left hand, and similarly $25\% - 20\% = 5\%$ of students, or 5 students, raise only their right hand. And 20% of students, or 20 students, raise both hands. So $5 + 10 + 20 = 35$ students have at least one hand raised, leaving $100 - 35 = \boxed{65}$ students without any hands raised.

27. This question is essentially asking for the surface area of the pancakes. The area of the top face of the pancakes is $6^2 \times \pi$ because each smaller radius pancake is within the boundaries of the 6 inch one. Now, looking at the side of the pancake, the area is $2r\pi \times 1 = 2r\pi$ where r is the radius of the pancake. In total, the side area is $2 \times 2 \times \pi + 2 \times 4 \times \pi + 2 \times 6 \times \pi = 24\pi$. In total, the surface area is $36\pi + 24\pi = 60\pi$. Therefore, the answer is $\boxed{60}$.

28. Notice that the square consists of two pairs of two congruent regions. Therefore, the combined area of the smallest and largest regions is equal to half the area of the square, which is $\frac{1}{2}$. Since the smallest region makes up $\frac{20}{20 + 23} = \frac{20}{43}$ of the total area of the smallest and largest regions, the smallest region has area $\frac{1}{2} \times \frac{20}{43} = \frac{10}{43}$. Therefore, the desired answer is $10 + 43 = \boxed{53}$.

29. Observe that every face of every $1 \times 1 \times 1$ cube is in one of two positions: touching a drop of glue, or facing the outside of the $4 \times 4 \times 4$ cube. There are $6 \times 4^2 = 96$ faces on the outside of the $4 \times 4 \times 4$ cube and $6 \times 4^3 = 384$ faces in total, so $384 - 96 = 288$ faces are touching a drop of glue. Since every drop of glue can be associated with 2 faces touching a drop of glue, there must be $288 \div 2 = \boxed{144}$ drops of glue.

30. First, notice that the fourth, sixth, and eighth leg will all have their numbers written in blue since the numbers on those legs will always be even and greater than two. We also know that the second leg will not have numbers of all the same color since 2 is not composite. The rest of the legs we can check manually. For the first leg, the numbers written are 1, 9, 17, 25, 33. Since 1 and 17 are not composite but the rest of the numbers are, this leg has two colors. The numbers on the third leg are 3, 11, 19, 27, 35 some of which are composite some of which are not. The fifth leg has numbers 5, 13, 21, 29 some of which are composite some of which are not. And finally, the seventh leg has the numbers 7, 15, 23, 31 some of which are not. Therefore, there are only $\boxed{3}$ legs with numbers of all the same color written on it.

31. For Alice, notice that the largest value she can get is $1 + 2 \times 3 \times 4$. This is because multiplication leads us to the largest value, and multiplying by 1 does not increase the value, so we change that to +. This gives us a maximum of 25. Alice's least value is $1 - 2 \times 3 \times 4 = -23$. On the other hand, Bob's largest value can be found to be $1 + 2 \times 3 \times 4 = (1 + 2) \times 3 \times 4 = 36$. This is because we want to utilize the fact that Bob does addition first. Then, his smallest value is $1 - 2 - 3 \times 4 = -16$, because Bob needs to have a large negative which he can achieve by doing subtraction first. So, we either have that the difference is $|25 - (-16)| = 41$, or $|36 - (-23)| = 59$. This is greater than 41, so the answer is $\boxed{59}$.

32. We figure out what digits work, and then calculate how many ways there are to rearrange those digits. We note that in the example given, 5255525, there are two 2's and five 5's. So considering other possibilities, there are seven 7's, one 1 and six 6's, two 2's and five 5's, three 3's and four 4's, and finally, one 1, one 2, and one 4. For each case, we can reorder the digits as needed.
For 7777777, there's 1 ordering.
For 1666666, there's $\binom{7}{1} = 7$ orderings.
For 2255555, there's $\binom{7}{2} = 21$ orderings.
For 3334444, there's $\binom{7}{3} = 35$ orderings.
For 1224444, there's $\frac{7!}{4!2!} = 105$ orderings.
In total, there are $1 + 7 + 21 + 35 + 105 = \boxed{169}$ numbers.

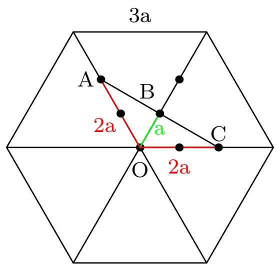
33. We proceed by casework based on the type of isosceles triangle. Suppose one of the equal sides of the triangle is on the x-axis. Now, we can do casework depending on what the side length of this triangle is. If it is 1, there are 8 of such triangles, and if it is 2, there are 6, and if it is 3, there are 4 of such triangles, and if the side length is 4, there are 2. Thus, there are $8+6+4+2 = 20$ of such triangles. Our second case is if the base (the side not equal to the either two) of the isosceles triangle falls on the x-axis. Therefore, the length of the base must be even, in order to have the third vertex be on a coordinate point with a x coordinate directly centered. So, we begin having a base of length 2. There are 3 such segments, and each have 4 triangles corresponding to the segment, making 12 triangles total. If the length of the base is 4, there is only one such base, and 4 triangles. So, there are $12 + 4 = 16$ triangles in this case. In total, we have $\boxed{36}$ isosceles triangles.

34. It is guaranteed that the 19 people who did not win the tournament lost exactly 5 games each. Also, the winner of the tournament lost at most 4 games. Therefore, the number of times some player lost some game is at most $19 \times 5 + 4 = 99$. Since every game yields precisely one loser, there must be a maximum of $\boxed{99}$ games played.

35. The horizontal line intersects the vertex of the parabola. Therefore, $b = 4$. The x-coordinates of the other two intersection points can be solved with $b = ax^2 - b$ or $x = \pm\sqrt{\frac{2b}{a}} = \pm\frac{2\sqrt{2}}{\sqrt{a}}$. The distance between these two points is $\frac{4\sqrt{2}}{\sqrt{a}} = 4$ so $a = \boxed{2}$.

36. The circumference of the circle in kilometers is $2\pi \times 6 = 12\pi$. Masha walked 8π kilometers, which is $\frac{2}{3}$ of the circumference. Masha's path back home is one side of an equilateral triangle that is inscribed in the circular park. The side length of this triangle is $6 \times \frac{3}{2} \times \frac{2}{\sqrt{3}} = 6\sqrt{3}$ kilometers. $6 + 3 = \boxed{9}$.

37. The diagram shows one third of the trisection.



Notice that the distances between the selected point and the center alternate between a and $2a$ where $3a$ is the side length of the hexagon. Since the formed line segments are 60 degrees apart (e.g. $\angle AOB = 60°$), the triangle formed by taking two adjacent selected points and the center is a 30-60-90 triangle (e.g. $\triangle AOB$). Therefore, every selected point a away from the center is collinear with the selected point before and after (e.g. A, B, C are collinear). The trisection is actually a equilateral triangle with side length $2a\sqrt{3}$. The ratio of the area of the hexagon to the area of the trisection is $\frac{(3a)^2 \cdot 6}{(2a\sqrt{3})^2} = \frac{9 \cdot 6}{4 \cdot 3} = \frac{9}{2}$. Since the area of the hexagon is 36, the area of its trisection is $4 \cdot 2 = \boxed{8}$.

38. All square numbers will have a units digit as either of $1, 4, 5, 6$, or 9. As we already know 1 is perplexing, and no other one-digit squares have a 1 in the units digit, we need to find the first two 4-digit squares which have a 4 in the units digit. 1024 is perplexing, as $32 \times 32 = 1024$, and 1444 is perplexing, as $38 \times 38 = 1444$. Therefore, our answer is $1 + 1024 + 1444 = \boxed{2469}$.

39. We can realize that if the least common multiple is contained in the subset, then the only other elements of the subset would be divisors of the least common multiple. For example, if we take the number 12, its divisors are $1, 2, 3, 4, 6, 12$. 12 must be in the subset, but the remaining 5 numbers can be in it or not, so there are 2^5 of such subsets for the largest element being 12. For all the least common multiples that are primes, there would be 2 ways to create this subset. Since there are 7 primes from 1 to 17, these comprise of $2 \cdot 7 = 14$ of the subsets. For the remaining subsets, there would be $1 + 2^2 + 2^3 + 2^3 + 2^2 + 2^3 + 2^5 + 2^3 + 2^3 + 2^4 = 97$. When added to 14, this would be $97 + 14 = \boxed{111}$.

40. Notice that the number $555...555_6$ with 2023 occurrences of the digit 5 is equal to $6^{2023} - 1$ in base 10. In addition, notice that $6^{2023} = 6 \cdot (6^2)^{1011} = 6000...000_{36}$, where there are 1011 occurrences of the digit 0. If we let D denote the digit for 35 in base 36, we then have that $6^{2023} - 1 = 5DDD...DDD_{36}$, where there are 1011 occurrences of the digit D. We want the digit sum of this base 36 integer, which is $5 + 1011 \cdot 35 = \boxed{35390}$.