Answer Key

Name: GRADE 7 KEY **ID**: GRADE 7 KEY

1 . 1095	11 . 7	21 . 56	31 . 56
2 . 2	12 . 28	22 . 16	32 . 22
3 . 120	13 . 474	23 . 100800	33 . 34
4 . 109	14 . 60	24 . 33	34 . 4
5 . -8090	15 . 252	25 . 3	35 . 3
6 . 36	16 . 2	26 . 28	36 . 688
7 . 11	17 . 9	27 . 4	37 . 11
8 . 17	18 . 90	28 . 32	38 . 16
9 . 5040	19 . 48	29 . 49	39 . 14
10 . 12	20 . 4	30 . 50	40 . 84

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4

Total Score:

Joe Holbrook Memorial Math Competition

7th Grade

October 22, 2023

1. $3 \times 365 = \boxed{1095}$.

2. Using order of operations, $(1 + 2 \times 3) - (1 \times 2 + 3) = (1 + 6) - (2 + 3) = 7 - 5 = \boxed{2}$.

3. There are 20 turtles, so there are $2 \cdot 20 = 40$ lizards, meaning that there are $3 \cdot 40 = \boxed{120}$ snakes.

4. Since Bob's number is odd, all odd squares can be disregarded, as 3 times an odd square + 1 is always even. Consider the first several even squares: 4, 16, 36. The numbers generated by multiplying them by three and adding 1 are 13, 49, 109. Since it is known that 13 is not Bob's number, and 49 isn't prime, the first odd prime would be $\boxed{109}$.

5. We write $2023 \cdot 2024 = (2021 + 2)(2022 + 2) = 2021 \cdot 2022 + 2 \cdot 2022 + 2021 \cdot 2 + 4$, so the answer is $-2 \cdot (2022 + 2021) + 4 = \boxed{-8090}$.

6. Carl has 3 options for shirts, 4 options for pants, and 3 options for hats (since he can wear one of the 2 hats or wear no hat). Therefore, he has $3 \times 4 \times 3 = \boxed{36}$ combinations of clothes he can wear.

7. Consider the worst-case scenario. In this scenario, Bob selects 10 different mittens(one of each color), but if he picks one more, then he must have a mitten that matches with another. Thus, Bob must pick $\boxed{11}$ mittens.

8. This can be represented as $\sqrt{3(x + 10)} + 8 = x$. Rearranging and squaring yields
$$3(x + 10) = x^2 - 16x + 64 \implies x^2 - 19x + 34 \implies (x - 2)(x - 17) = 0$$
. The answer is $x = \boxed{17}$, as plugging 2 into the expression does not yield two again.

9. The circumference of a wheel is $(2\pi) \times (\frac{3}{\pi}) = 6$ feet, so the bus travels $14 \times 6 = 84$ feet per second. In one minute, the bus therefore travels $84 \times 60 = \boxed{5040}$ feet.

10. The angles being equal means that the triangle is isosceles, meaning that it is symmetric and two of the sides (specifically AB and AC) have the same length. So, the length of the final side must be either 7 or 5, for a total of $\boxed{12}$.

11. The divisibility rule for 9 is that the sum of digits must be a multiple of 9. $8 + 2 + 7 + 3 = 20$, and the only multiple of 9 that is between 20 and 30 is 27, so the missing digit must be $27 - 20 = \boxed{7}$.

12. The area of the original 6×6 square of origami paper is $6 \times 6 = 36$ square units. The ripped origami paper has two 2×2 squares removed from its opposite corners. Each of the two 2×2 squares accounts for 4 square units of removed paper, for a total of 8 square units removed. So the area is now $36 - 8 = \boxed{28}$ square units.

13. Notice that we can factor the equation to get $q(p + r) = 243$. Since q is prime and the only prime factor of 243 is 3, we have that $q = 3$. Therefore, we have $q + r = 81$. Notice that q and r can't both be odd, meaning that either $q = 2$ and $r = 79$ or $r = 2$ and $q = 79$. Whichever case we choose, we get that the three primes are 3, 2, and 79, meaning that the answer is $3 \cdot 2 \cdot 79 = \boxed{474}$.

14. This question is essentially asking for the surface area of the pancakes. The area of the top face of the pancakes is $6^2 \times \pi$ because each smaller radius pancake is within the boundaries of the 6 inch one. Now, looking at the side of the pancake, the area is $2r\pi \times 1 = 2r\pi$ where r is the radius of the pancake. In total, the side area is $2 \times 2 \times \pi + 2 \times 4 \times \pi + 2 \times 6 \times \pi = 24\pi$. In total, the surface area is $36\pi + 24\pi = 60\pi$. Therefore, the answer is $\boxed{60}$.

15. If Anthony's box is 216 cubic feet, a side of his box is $\sqrt[3]{216}$ feet, or 6 feet long. The radius of the balloon's volume will be half of this or 3 feet, and the volume of the balloon $= \frac{4}{3}\pi(3)^3 = 36\pi$. Thus, the difference between the box's volume and the balloon's volume will be $216 - 36\pi$, and our answer will be $216 + 36 = \boxed{252}$.

16. The pattern for the units digit of 7^x follows $7, 9, 3, 1, 7\ldots$ Since these digits occur in a cycle of 4, we must find the remainder when 2023 is divided by 4. This remainder is 3, so the units digit of 7^{2023} is 3. The pattern for 9^x is $9, 1, 9, \ldots$ So, we should check whether the exponent is even or odd to see if it is 9 or 1. Since the exponent is odd, the units digit of 9^{2023} is 9. Thus, we can sum these two numbers for $3+9 = 12$, giving the units digit as $\boxed{2}$.

17. The circumference of the circle in kilometers is $2\pi \times 6 = 12\pi$. Masha walked 8π kilometers, which is $\frac{2}{3}$ of the circumference. Masha's path back home is one side of an equilateral triangle that is inscribed in the circular park. The side length of this triangle is $6 \times \frac{3}{2} \times \frac{2}{\sqrt{3}} = 6\sqrt{3}$ kilometers. $6 + 3 = \boxed{9}$.

18. If Elena rolls a 1, Lila can roll either a 3, 4, 5, or 6. There is a $\frac{1}{6}$ chance Elena rolls a 1 and a $\frac{4}{6}$ chance Lila rolls a 3, 4, 5, or 6 which is a $\frac{1}{6} \cdot 46 = \frac{4}{36}$ chance of this case. If Elena rolls a 2, Lila can roll either a 4, 5, or 6, which makes this case a $\frac{1}{6} \cdot \frac{3}{6} = \frac{3}{36}$ possibility. If Elena rolls a 3, Lila can roll a 5 or 6, which gives this case a $\frac{1}{6} \cdot \frac{2}{6} = \frac{2}{36}$ possibility. Finally, if Elena rolls a 4, Lila can roll a 6, which gives a $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ possibility. If Elena rolls a 5 or 6, there is no way for Lila to have a roll that is at least two higher than Elena's. So, all the cases in which Lila gets her preferred outcome have a possibility of $\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{10}{36} = \frac{5}{18}$. So, $m = 5, n = 18$, and $mn = \boxed{90}$.

19. The area of the smaller circle is 4π, while the area of the larger circle is 196π. Subtracting these areas will give the area outside of the smaller circle but still inside the larger circle, which is 196π - $4\pi = 192\pi$. The ratio of this area to the area of the smaller circle, is $\frac{192\pi}{4\pi} = \boxed{48}$.

20. Notice that since the first two terms of the arithmetic sequence are x and y, we have that the common difference of the sequence must be $y-x$. Therefore, we must have $2y-x = x+y$ and $2y = xy$. Rearranging terms in the first equation gives $y = 2x$, and substituting this into the second equation gives $4x = xy$. Since x is nonzero, we divide both sides by x to get that $y = \boxed{4}$.

21. Since the last digit of $\underline{AB} + \underline{B9}$ is A, it follows that A is one less than B. Also, the sum of A, B, and 1 must equal 16, implying that A equals 7 and B equals 8. The product of 7 and 8 is $\boxed{56}$.

22. Notice that because the hexagon is equiangular, each angle must be 120 degrees. Now, we can inscribe this hexagon in a rectangle, with the opposite sides of 2 and 4 lying on opposite sides of the rectangle. To find the area of the hexagon, we can subtract the area of the four corner triangles, which are $30 - 60 - 90$ triangles because of the measure of each angle in the hexagon. The triangles with hypotenuse 4 will have area $2\sqrt{3}$ each, and the triangles with hypotenuse 2 have area $\frac{\sqrt{3}}{2}$ each. The total rectangle will have an area of $(2\sqrt{3}+\sqrt{3})(1+4+1) = 18\sqrt{3}$. Subtracting the triangle areas, we obtain $18\sqrt{3}-4\sqrt{3}-\sqrt{3} = 13\sqrt{3}$, giving an answer of $13 + 3 = \boxed{16}$.

23. There are $10! = 10 \cdot 9 \cdot 8 \cdot \cdots \cdot 1$ ways to arrange 10 objects, but we have to account for C and M each appearing three times. There are $3! = 6$ ways to arrange just the Cs or just the Ms, so we need to divide by $6 \cdot 6 = 36$ (otherwise we count each arrangement 36 times). So, the answer is

$$\frac{10!}{36} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 3 \cdot 2} = \boxed{100800}.$$

24. Let the point where $\overrightarrow{\mathrm{AO}}$ meets $\overline{BC}$ be X. By the problem statement, $\angle OXC = \angle OXB = 90°$. By Hypotenuse-Leg congruence, $\triangle OXC$ is congruent to $\triangle OXB$, so $XB = XC = \frac{8}{5}$. By the Pythagorean theorem, $OX = \sqrt{2^2 - (\frac{8}{5})^2} = \frac{6}{5}$.

Note that there are two possible configurations for this problem, one where the triangle is acute and one where it is obtuse. This is because the altitude from O to $\overline{BC}$ meets the circle at two points.

If the triangle is acute, $AX = 2 + \frac{6}{5} = \frac{16}{5}$, so the area, given by $\frac{1}{2} \cdot b \cdot h$, is $\frac{1}{2} \cdot \frac{16}{5} \cdot \frac{16}{5}$, which equals $\frac{128}{25}$, or $5\frac{3}{25}$. $5 + 3 + 25 = 33$.

If the triangle is obtuse, $AX = 2 - \frac{6}{5} = \frac{4}{5}$, so the area is $\frac{1}{2} \cdot \frac{16}{5} \cdot \frac{4}{5}$, which equals $\frac{32}{25}$, or $1\frac{7}{25}$. $1+7+25 = 33$.

In both cases, the relevant sum of $x + y + z$ is equal to 33, so the answer is $\boxed{33}$.

25. First, notice that the fourth, sixth, and eighth leg will all have their numbers written in blue since the numbers on those legs will always be even and greater than two. We also know that the second leg will not have numbers of all the same color since 2 is not composite. The rest of the legs we can check manually. For the first leg, the numbers written are 1, 9, 17, 25, 33. Since 1 and 17 are not composite but the rest of the numbers are, this leg has two colors. The numbers on the third leg are 3, 11, 19, 27, 35 some of which are composite some of which are not. The fifth leg has numbers 5, 13, 21, 29 some of which are composite some of which are not. And finally, the seventh leg has the numbers 7, 15, 23, 31 some of which are not. Therefore, there are only $\boxed{3}$ legs with numbers of all the same color written on it.

26. If you know about combinations, this is just $\binom{9}{3} - \binom{8}{3} = 84 - 56 = \boxed{28}$. If not, you can also think it through. In the nonagon case, for example, there are 9 choices for the first vertex, 8 for the next, and 7 for the last, but you have to divide by $3! = 6$ since the order does not matter.

27. Note that the resulting shape is another regular tetrahedron, so it will be similar to the original. The area of any individual face must have decreased to $\frac{4}{9}$ of the original, then, and since area is proportional to side length squared (in a pair of similar triangles), the new side length is $\frac{2}{3}$ of the original. The height of the tetrahedron is proportional to the side length, so $1 - \frac{2}{3} = \frac{1}{3}$ of it must have been cut off, and the answer is $\boxed{4}$.

28. If O_1 and O_2 are the centers of the circles, and the circles intersect at A and B, then $\triangle O_1AO_2$ and $\triangle O_1BO_2$ are equilateral. Thus, $\angle AO_1B = \angle AO_2B = 120°$. So the perimeter of the region of points covered by at least one of the two circles consists of two-thirds the perimeter of one circle and two-thirds the perimeter of the other circle. The total perimeter is therefore four-thirds the perimeter of one circle, or $\frac{4}{3} \times 2\pi(\frac{12}{\pi}) = \boxed{32}$.

29. Either he makes 8 northeast steps and 1 southwest step or he makes 7 northeast steps, 1 northwest step, and 1 southeast step, since he needs 7 northeast steps just to get to the opposite corner as directly as possible. The first case has 7 possibilities (the SW step cannot be the first or last, since then Kelvin would go off the chessboard). For the second case, assume he goes southeast before going northwest; we will double the answer at the end. There are six places where he can leave the direct path (between any two northeast steps), so if he returns immediately there are 6 options and if he returns later there are $\binom{6}{2} = 15$ options. So, in total, the answer is $7 + 2(6 + 15) = \boxed{49}$.

30. Let A be the point $(0, 0)$ and B be the point $(6, 8)$. Using the distance formula, we know that $AB = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

Now, let's consider the orientation of the square. We have two cases to consider.

Case 1: A and B are adjacent vertices of the square. Then, AB is the side length. The area of the square is then simply the square of the side length, or $10^2 = 100$ square units.

Case 2: A and B are diagonally opposite vertices of the square. In a square, the diagonals have the same length, so the length of the diagonal of the square is 10 units. The side length of the square is then the diagonal divided by $\sqrt{2}$ (as the diagonal, side, and a perpendicular from one vertex to the diagonal form a 45-45-90 triangle). Thus, the side length is $\frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ units, and the area of the square is $(5\sqrt{2})^2 = 50$ square units.

So, the difference between the maximum and minimum possible area of the square is $100 - 50 = \boxed{50}$ square units.

31. Note that (c) implies Daniel sent 30 or 56 messages, but (d) implies that he sent 77 messages. So, one of them must be false and the other statements are true. (b) means that Andrea and Jeremy sent 10 and 30 messages in some order; either way, (e) implies Anthony must have sent 77 messages. That leaves only $\boxed{56}$ for Daniel.

32. We can work recursively. Let p_n be the maximum number of pieces that can be created with n cuts. Notice that to maximize the number of pieces created after the $n+1$th cut, the $n+1$th line must cross all other n lines inside the circle, but not pass through any pairwise intersections between lines.

    As the $n+1$th line will pass through each of the n line segments, it gets split into $n+1$ segments. Each of these segments splits a piece that was already formed by n cuts in two, thus adding $n+1$ pieces.

    So, the base case must be with $n = 1$, which can only split the pizza into two slices. Thus, $p_2 = p_1 + 2 = 3$. Continuing, we obtain that $p_6 = 2+2+3+4+5+6 = \boxed{22}$.

33. Notice that pushing the first button twice does exactly the same thing as pushing the second button once. So, no matter what, Mark can only reach the numbers 1, 5, 17, 53, 161, 485, and 1457 before going over 2023. Each number can only be reached from the two before it, so the numbers of ways to reach each of the above numbers are 1, 1, 2, 3, 5, 8, and 13. (For example, Mark can either push the first button on 485 (8 possibilities) or the second on 161 (5 possibilities), for a total of 13; this is the Fibonacci sequence.) Mark can press the second button on 485 or either button on 1457 to make the calculator error, so the answer is $8 + 13 + 13 = \boxed{34}$.

34. Taking the diagonal cross section of the cube, we get a rectangle with length $2\sqrt{2}$ and width 2. The circle sits in the middle tangent to the top and the bottom. Setting the length outside the circle to x. We use Power of a Point to get $2 = x \cdot \sqrt{6}$. We then get a ratio of $\dfrac{\frac{2}{\sqrt{6}}}{\sqrt{6}} = \dfrac{1}{3}$ giving an answer of $\boxed{4}$.

35. Notice that the expression $x^4+8x^3+24x^2+29x+10$ is $(x+2)^4-3(x+2)$. So, we have that $(x+2)^4 = 3(x+2)$, from which we can divide by $x+2$ on both sides (since we're given $x \neq -2$) to get $(x+2)^3 = \boxed{3}$.

36. Notice that E being a divisor of BEE, EBE, or EEB is equivalent to E being a divisor of B00, B0, or B, respectively. Thus, E is a divisor of B00 but not B0, implying that either E equals 4 and B is odd, or E equals 8 and B is either 2 or 6. So the only options are:

    $$\text{BEE} = 144,\ 344,\ 544,\ 744,\ 944,\ 288,\ 688.$$

    The product of the digits of these numbers, respectively, are:

    $$(144, 16),\ (344, 48),\ (544, 80),\ (744, 102),\ (944, 144),\ (288, 128),\ (688, 384).$$

    The only product of digits that matches "SET" is the one corresponding to the secret code being $\boxed{688}$.

37. Let the extension of CE intersect the extension of AB at F. By doing some angle chasing, we find that $\triangle BCD$ is isosceles, giving $CD = 22$. In addition, we also get that $\triangle CEF$ is isosceles, giving $BF = 22$. This means that $AF = 2$. Finally, we also get that $\triangle AFE \sim \triangle DCE$, giving that $DE = \dfrac{CD \cdot AE}{AF} = \dfrac{22}{2} = \boxed{11}$.

38. Rewrite $2xy + 7x - 8y = 0$ as follows:

    $$(x-4)(2y+7) + 28 = 0 \implies (x-4)(2y+7) = -28$$

    Since both $x-4$ and $2y+7$ are always integers, $(x-4,\ 2y+7)$ must be a factor pair of -28. Furthermore, notice that $2y+7$ is always odd. So, the possible factor pairs are $(-28, 1)$, $(-4, 7)$, $(4, -7)$, and $(28, 1)$. These correspond to $x = -24, 0, 8$, and 32, respectively. So, the sum of all integers x is $\boxed{16}$.

39. First we note that when placing a plus sign, the value of the expression (mod 9) is invariant. This is because the sum of the digits of a number has the same remainder when divided by 9 as the number itself. Therefore, n must be congruent to 6 (mod 9).

    6 and 15 can be achieved with one 111 and either one 11 and one 1 or 12 ones. 24 can be achieved with 11 elevens and 2 ones. All other numbers congruent to 6 (mod 9) between 33 and 123 can be achieved by exchanging an 11 for 11 ones.

    In total, all of the numbers congruent to 6 (mod 9) between 6 and 123, inclusive, can be reached, giving $\boxed{14}$ in total.

40. Notice that the condition is equivalent to $(p+1)(p+q-1) = 3984 = 2^4 \cdot 3 \cdot 83$. Notice that $p = 2$ gives $q = 1327$, which is a prime, but p is not an odd prime so this is invalid. Also, $q = 2$ doesn't give an integer value for p, so therefore, p and q are both odd primes.

    Notice that because of this, $p + 1 < p + q - 1$ and $p + 1$ must be even while $p + q - 1$ is odd.

    Now, we can do casework on the values of $p + 1$ and $p + q - 1$:

    Case 1 ($p + 1 = 48$, $p + q - 1 = 83$): This case gives $p = 47$ and $q = 37$, which works.

    Case 2 ($p + 1 = 16$, $p + q - 1 = 249$): This case gives $p = 15$, which doesn't work.

    Therefore, $p = 47$ and $q = 37$, meaning that $p + q = 47 + 37 = \boxed{84}$.