Joe Holbrook Memorial Math Competition

4th Grade

October 20, 2024

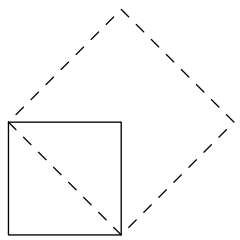
General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is $2 \times 2 + 0 + 4 \times 4 \times 4 + 2 \times 2$?
2. What number ♠ satisfies the equation ♠ + ♠ + ♠ = 27?
3. What number is halfway between 11 and 111 on the number line?
4. Compute $(2 + 0 + 2 + 4) \times (20 + 24)$.
5. Brian is taking his JHMMC test, which is 75 minutes. He falls asleep for 20% of the test but stays awake for the rest. How many minutes was Brian awake?
6. How many positive multiples of 11 are less than 200?
7. Jeremy goes to math class for 50 minutes every school day. If school runs for 180 total days, how many hours will Jeremy spend in math class this year? (There are 60 minutes in an hour.)
8. If 50 students completely fill 2 buses, how many buses are needed to fit 289 students?
9. Every day, Tony the Tree's height doubles from the previous day. Two days ago, Tony was 13 feet tall, and yesterday, he was 26 feet tall. In feet, how tall will Tony the Tree be tomorrow?
10. At the start of the school year, Michael buys ten stacks of 90 sticky notes each. He uses 3 sticky notes each day in school for the entire school year, which is 180 days long. After the school year, how many sticky notes does he have left?
11. One purlunkle equals 10 inches. How many purlunkles equals 20 feet? (Note that there are 12 inches in a foot.)
12. Find the sum of the distinct prime factors of 2024.
13. Arnav is practicing archery! He has a 10% chance of hitting a bullseye on each shot, but his friend will only give him a cookie if he hits a bullseye on both of his next two shots! If the chance Arnav gets a cookie can be written as $\frac{m}{n}$ in simplest form, what is $m + n$?
14. What is the remainder when $41^2 - 4$ is divided by 7?
15. Increasing the length of a rectangle by 25 percent and the width by x percent doubles the rectangle's area. Find x.
16. A palindrome is a number which reads the same forwards and backwards. For example, both 22 and 494 are palindromes. Two positive integers x and $x + 13$ are two and three digit palindromes, respectively. Compute x.
17. Caleb has a dollar worth of pennies and a dollar worth of dimes. He accidentally drops his coins, and can only pick up exactly half of the coins he has before the rest fall down the drain. What is the most money, in cents, that Caleb could have after he dropped his coins?
18. Alex picks socks out of his sock drawer every morning, one at a time. However, Alex does not know how many pairs of socks he owns! This morning, he wants to pick 8 individual socks from his drawer. If Alex wants to guarantee that he picks out at least one pair of socks, what is the greatest number of distinct pairs of socks that could be in the drawer?
19. The three angles of a triangle form a geometric progression with the value of the common ratio being 3. Let x be the smallest angle of this triangle. Find the greatest integer less than x.
20. Two squares are placed such that a diagonal of one square is a side of the other square. How many times bigger is the area of the larger (dashed) square than the smaller square?



21. The mean of 8 integers is 17. If the maximum value is 34, and the median is 20, what is the absolute value of the minimum value of the smallest number in the list?

22. It takes Jeremy 30 minutes to walk from his house to Andrea's house. If he walks half the distance to Andrea's house, and bikes the rest, it takes him 20 minutes total. If Jeremy's walking speed is two miles per hour, in miles per hour, how fast does he bike?

23. Define $a\Delta b = a \cdot (a + b)$. Compute $3\Delta(4\Delta5)$.

24. Dolly has a hexagon $ABCDEF$, and she can build identical houses on each vertex of the hexagon. But, she does not want houses on adjacent vertices. If Dolly wants to build at least one new house, how many different ways can she build new houses on her land?

25. What is the fewest number of coins required to make 49 cents if we can only use pennies, nickels, dimes, and quarters?

26. Andy visits his grandmother today, a Saturday, and keeps doing so once every 17 days. How many days from now will he visit his grandmother on a Sunday?

27. The country of Mathlandia uses a weird currency system: 3 toves equal 14 borogoves, 2 borogoves equal 5 jabberwocks, and 6 mome equal 1 tove. If a mome are b jabberwocks, where a, b share no common factors, what is $a + b$?

28. Cristiano is trying to qualify for a soccer tournament. Currently, he has won 40% of the 20 games he has played. If Cristiano needs to win at least 75% of his games to qualify for the soccer tournament, how many more games does he need to play to qualify, assuming he wins all of the games?

29. Snorlax places the four integers $1, 2, 3, 4$ into a 2×2 grid. He finds the product of the numbers in the top row and bottom row, as well as the left and right column. Snorlax then adds all these numbers together. What is the maximum possible sum Snorlax could have?

30. Alice wants to collect some nonzero number of hexagons and pentagons. She wants the total number of sides of the hexagons to equal the total number of sides of the pentagons. What is the minimum number of shapes she needs in total?

31. What is the minimum number of people that must be at a baseball game to guarantee that there are two people with the same initials?

32. Sabrina and Amy are crossing the street to the opposite corner of an intersection. Sabrina crosses one crosswalk before crossing the perpendicular crosswalk, while Amy cuts across the intersection diagonally. If they both start and end in the same places, one crosswalk is 16 feet, and the perpendicular crosswalk is 12 feet, how many more feet does Sabrina have to walk than Amy?

33. Two congruent isosceles right triangles are placed (without overlap) such that a nonzero fraction of the perimeter of each triangle is shared. The result is a polygon with n sides. What is the sum of all possible values of n?

34. Suppose we have $\triangle ABC$ such that $AB = 5$, $BC = 8$, and $\triangle DEF$ such that $DE = 15$, $EF = 24$, and $\angle ABC$ has the same angle measure as $\angle DEF$. If the area of $\triangle DEF$ is 18, find the area of $\triangle ABC$.

35. Triangle ABC has right angle A, $BC = 60$, and $AB = 30$. Points P and Q lie on BC. Segments AP and AQ trisect angle ABC. Compute the length of PQ.

36. Chris has invested in some state of the art crayons for his art project. He has red, blue, and green crayons. He tells you the following information:

    (a) All but 7 of my crayons are red.

    (b) All but 3 of my crayons are blue.

    (c) All but 8 of my crayons are green.

    How many total crayons does he have?

37. Let $x!$ be the function that finds the product of all positive integers less than or equal to x. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the remainder when the sum $(1!)^2 + (2!)^2 + (3!)^2 + ... + (100!)^2$ is divided by 100.

38. Find the smallest 6-digit number $\overline{ABCDEF}$ (digits not necessarily distinct) such that $2 \mid \overline{AB}$, $3 \mid \overline{ABC}$, $4 \mid \overline{ABCD}$, $5 \mid \overline{ABCDE}$, $6 \mid \overline{ABCDEF}$. (The notation $n \mid x$ says that x is divisible by n.)

39. Distinct prime numbers p, q, and r satisfy the equation that $pq + qr + pr + pqr = 617$. Find the value of $p + q + r$.

40. How many fractions of the form $\dfrac{x}{x+72}$ with $1 \leq x \leq 2024$ are in simplest terms?