Joe Holbrook Memorial Math Competition

5th Grade

October 20, 2024

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.

- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.

- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper

- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.

- You do not need to write units in your answers.

- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What number ♠ satisfies the equation $♠ + ♠ + ♠ = 27$?

2. How many full 473 milliliter bottles of water can you pour into a 5 liter bucket without spilling? (Note: There are 1000 milliliters in 1 liter.)

3. How much bigger is $2024 + 2023$ than $2024 - 2023$?

4. Compute $(2 + 0 + 2 + 4) \times (20 + 24)$.

5. What is the sum of 24×25 and 25×26?

6. Anoushka buys a pizza. Brandon has half of the whole pizza, and Carlin has a third of what's left. Anoushka has the last two slices. How many slices did the pizza have?

7. How many two-digit numbers whose digits are both prime are prime?

8. If 50 students completely fill 2 buses, how many buses are needed to fit 289 students?

9. Every day, Tony the Tree's height doubles from the previous day. Two days ago, Tony was 13 feet tall, and yesterday, he was 26 feet tall. In feet, how tall will Tony the Tree be tomorrow?

10. Jeremy is randomly selecting letters from the alphabet. After he selects a letter, he cannot select it again. How many letters must he pick to guarantee that he picks at least one vowel (a, e, i, o, u, or y)?

11. A duck family consists of a mother duck, a father duck, and 3 baby ducks. Given that every duck in a pond belongs to a duck family, how many baby ducks are in a pond of 120 ducks?

12. One purlunkle equals 10 inches. How many purlunkles equals 20 feet? (Note that there are 12 inches in a foot.)

13. Arnav is practicing archery! He has a 10% chance of hitting a bullseye on each shot, but his friend will only give him a cookie if he hits a bullseye on both of his next two shots! If the chance Arnav gets a cookie can be written as $\dfrac{m}{n}$ in simplest form, what is $m + n$?

14. Sabrina and Amy are crossing the street to the opposite corner of an intersection. Sabrina crosses one crosswalk before crossing the perpendicular crosswalk, while Amy cuts across the intersection diagonally. If they both start and end in the same places, one crosswalk is 16 feet, and the perpendicular crosswalk is 12 feet, how many more feet does Sabrina have to walk than Amy?

15. At BCA Math Team, we offer free donuts to all members. Suppose there are 80 members, and 90% of them eat one donut, 5% eat two donuts, and 5% eat zero donuts. What is the average number of donuts eaten per member?

16. Alex picks socks out of his sock drawer every morning, one at a time. However, Alex does not know how many pairs of socks he owns! This morning, he wants to pick 8 individual socks from his drawer. If Alex wants to guarantee that he picks out at least one pair of socks, what is the greatest number of distinct pairs of socks that could be in the drawer?

17. Increasing the length of a rectangle by 25 percent and the width by x percent doubles the rectangle's area. Find x.

18. Gabriele starts drawing squares, each with an area of 4. None of the squares overlap or share an edge with each other. Once the total area of all of Gabriele's squares added together is 100, he stops. What is the total perimeter of all of Gabriele's squares?

19. Danny and Nikhil are mixing coffee with milk. Danny's flask has 400 mL of liquid, which is $\frac{3}{5}$ coffee. Danny steals Nikhil's 200 mL full cup of liquid and pours it into his flask. He finds out that his flask is now $\frac{1}{2}$ coffee. What percent of Nikhil's original mixture was coffee? (If the percent is $x\%$, report your answer as x.)

20. A triangle has vertices at $(0, 1)$, $(4, 1)$, and $(4, 4)$. What is the length of its longest side?

21. Caleb has a dollar worth of pennies and a dollar worth of dimes. He accidentally drops his coins, and can only pick up exactly half of the coins he has before the rest fall down the drain. What is the most money, in cents, that Caleb could have after he dropped his coins?

22. Andy visits his grandmother today, a Saturday, and keeps doing so once every 17 days. How many days from now will he visit his grandmother on a Sunday?

23. What are the last two digits of 2024^{2024}?

24. Snorlax places the four integers $1, 2, 3, 4$ into a 2×2 grid. He finds the product of the numbers in the top row and bottom row, as well as the left and right column. Snorlax then adds all these numbers together. What is the maximum possible sum Snorlax could have?

25. The three angles of a triangle form a geometric progression with the value of the common ratio being 3. Let x be the smallest angle of this triangle. Find the greatest integer less than x.

26. Jay has 400 units of wire and tries to form as many regular polygons with side length 1 as he can with the wire he has. Once a regular polygon is formed, he does not reuse the wire used to make that polygon. If each polygon he forms has a different number of sides, what is the maximum number of unique polygons he can form?

27. Chris has invested in some state of the art crayons for his art project. He has red, blue, and green crayons. He tells you the following information:

    (a) All but 7 of my crayons are red.
    (b) All but 3 of my crayons are blue.
    (c) All but 8 of my crayons are green.

    How many total crayons does he have?

28. Michael replaced every digit in the sum of two numbers by the characters A, B, and C, as shown below. Each character represents a single digit from 0 to 9. What is the greatest possible numerical value of ACBA if we only know Michael's addition is correct?

$$\begin{array}{r} \text{BCA} \\ + \text{ABC} \\ \hline \text{ACBA} \end{array}$$

29. Alice wants to collect some nonzero number of hexagons and pentagons. She wants the total number of sides of the hexagons to equal the total number of sides of the pentagons. What is the minimum number of shapes she needs in total?

30. Of five positive integers, the second is triple the first, the third is 5 times the second, the fourth is 4 times the third, and the fifth is half the first. If the difference between a certain two of these integers is 37, find the sum of all the numbers.

31. Suppose that for some x, we have $3^{\frac{x}{2}} = 2$. Find 9^{2x+1}.

32. Let x be the answer to this question. Given that x is nonzero and y is half of x, find the average of y, $y + 23$, and $y + 46$.

33. Let $x!$ be the function that finds the product of all positive integers less than or equal to x. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the remainder when the sum $(1!)^2 + (2!)^2 + (3!)^2 + ... + (100!)^2$ is divided by 100.

34. Alex has a balloon with a volume of 40 cubic inches and a surface area of 32 square inches. When the balloon expands to 320 cubic inches, Alex observes that it keeps the same shape. What is the new surface area of the balloon?

35. A dog staring out the window barks every time an animal passes by. If more than one animals pass at the same time, the dog still barks once. Cats pass by every 3 minutes, rabbits pass by every 5 minutes, and deer pass by every 10 minutes. There are no other animals in the neighborhood. Assuming the dog can only see directly in front of it, how many times does it bark in 1 hour? (We start counting immediately after all 3 animals passed by at the same time).

36. Twenty-six people sit in a circle. At random, each person takes a marble from a jar with 24 indistinguishable green marbles, 1 blue marble, and 1 red marble. The person with the blue marble and the person with the red marble are not beside each other. How many distinct ways are there to distribute the marbles in this way?

37. Let m be the minimum value of
$$x^2 + 2xy + 2y^2 + 4y$$
over all real numbers x and y, and let a and b be the unique real numbers such that
$$a^2 + 2ab + 2b^2 + 4b = m.$$
Compute the value of mab.

38. Cam and Devin are playing a game. Cam randomly selects a number between 1 and 10 (inclusive) and Devin rolls a die, and whoever has the biggest number wins. If they tie, Cam chooses a new number and Devin re-rolls the die. They do this until someone wins. Given that the chance that Cam wins can be represented as $\dfrac{p}{q}$ when written in simplest form, find $p + q$.

39. Devin draws square $XMYC$ with side length 4. He picks a random point A inside of the square. If the probability that the area of triangle CAM is at least 4 can be expressed as $\dfrac{p}{q}$ in simplest form, find $p + q$.

40. Distinct prime numbers p, q, and r satisfy the equation that $pq + qr + pr + pqr = 617$. Find the value of $p + q + r$.