Joe Holbrook Memorial Math Competition

6th Grade

October 20, 2024

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What number $\spadesuit$ satisfies the equation $\spadesuit + \spadesuit + \spadesuit = 27$?

2. Pylypovych's Pizzeria is notorious for its impossible pizzas: each pizza is half a foot thick! Pylypovych wants to create a stack of pizzas as tall as the Empire State Building, which is 1,454 feet tall. How many pizzas does Pylypovych need?

3. Every day, Tony the Tree's height doubles from the previous day. Two days ago, Tony was 13 feet tall, and yesterday, he was 26 feet tall. In feet, how tall will Tony the Tree be tomorrow?

4. Compute $(2 + 0 + 2 + 4) \times (20 + 24)$.

5. How many positive multiples of 11 are less than 200?

6. At the start of the school year, Michael buys ten stacks of 90 sticky notes each. He uses 3 sticky notes each day in school for the entire school year, which is 180 days long. After the school year, how many sticky notes does he have left?

7. Jeremy is randomly selecting letters from the alphabet. After he selects a letter, he cannot select it again. How many letters must he pick to guarantee that he picks at least one vowel (a, e, i, o, u, or y)?

8. Arnav is practicing archery! He has a 10% chance of hitting a bullseye on each shot, but his friend will only give him a cookie if he hits a bullseye on both of his next two shots! If the chance Arnav gets a cookie can be written as $\frac{m}{n}$ in simplest form, what is $m + n$?

9. One purlunkle equals 10 inches. How many purlunkles equals 20 feet? (Note that there are 12 inches in a foot.)

10. Gabriele starts drawing squares, each with an area of 4. None of the squares overlap or share an edge with each other. Once the total area of all of Gabriele's squares added together is 100, he stops. What is the total perimeter of all of Gabriele's squares?

11. What is the fewest number of coins required to make 49 cents if we can only use pennies, nickels, dimes, and quarters?

12. Alex draws a square with side length s, and he draws a rectangle with side lengths $7s$ and $3s$. If the perimeter of his rectangle is 100, what is the area of his square?

13. Andy visits his grandmother today, a Saturday, and keeps doing so once every 17 days. How many days from now will he visit his grandmother on a Sunday?

14. Define $a\Delta b = a \cdot (a + b)$. Compute $3\Delta(4\Delta 5)$.

15. Today is Bear's birthday! Because of this, Bear goes to his favorite ice cream store, which only serves 7-scoop and 9-scoop ice cream cones. However, Bear is picky and wants exactly 100 scoops of ice cream. What is the least amount of ice cream cones Bear has to buy to have exactly 100 scoops of ice cream?

16. Increasing the length of a rectangle by 25 percent and the width by x percent doubles the rectangle's area. Find x.

17. Caleb has a dollar worth of pennies and a dollar worth of dimes. He accidentally drops his coins, and can only pick up exactly half of the coins he has before the rest fall down the drain. What is the most money, in cents, that Caleb could have after he dropped his coins?

18. A palindrome is a number which reads the same forwards and backwards. For example, both 22 and 494 are palindromes. Two positive integers x and $x + 13$ are two and three digit palindromes, respectively. Compute x.

19. Mythreya picks a random prime number less than 20. The probability it is even can be expressed as $\frac{p}{q}$ in simplest terms. What is $p + q$?

20. Let $f(n)$ be the sum of squares of the digits of n. Find $f(f(f(14))))$.

21. Dolly has a hexagon $ABCDEF$, and she can build identical houses on each vertex of the hexagon. But, she does not want houses on adjacent vertices. If Dolly wants to build at least one new house, how many different ways can she build new houses on her land?

22. What are the last two digits of 2024^{2024}?

23. Help! Thor is trying to guess my secret number. Given that it is between 70 and 79, all I can tell Thor is that my number is divisible by the number of factors it has. He is able to guess my number after that clue. What is it?

24. Alice wants to collect some nonzero number of hexagons and pentagons. She wants the total number of sides of the hexagons to equal the total number of sides of the pentagons. What is the minimum number of shapes she needs in total?

25. Suppose we have $\triangle ABC$ such that $AB = 5$, $BC = 8$, and $\triangle DEF$ such that $DE = 15$, $EF = 24$, and $\angle ABC$ has the same angle measure as $\angle DEF$. If the area of $\triangle DEF$ is 18, find the area of $\triangle ABC$.

26. From a deck of 10 cards labeled with the numbers 1 through 10, 5 cards are chosen in order without replacement. The probability that the cards are chosen in decreasing order can be represented by the fraction $\frac{a}{b}$ in simplest form. Compute $a + b$.

27. Define a sequence such that $a_1 = 4, a_2 = 4$, and for $n \geq 3$, $a_n = a_{n-1}a_{n-2}$. If a_8 can be written as 2^k for some whole number k, find k.

28. Andrea has a box of 10 marbles. Of them, 5 are red, 3 are green, and 2 are blue, and she randomly picks 3 marbles without replacement. If the probability that all 3 marbles are different colors is $\frac{m}{n}$ in simplest form, what is $m + n$?

29. Cam and Devin are playing a game. Cam randomly selects a number between 1 and 10 (inclusive) and Devin rolls a die, and whoever has the biggest number wins. If they tie, Cam chooses a new number and Devin re-rolls the die. They do this until someone wins. Given that the chance that Cam wins can be represented as $\frac{p}{q}$ when written in simplest form, find $p + q$.

30. Alice has 3 positive integers a, b, and c. She tells you 3 statements:

    (a) The mean of the numbers is 3.

    (b) The median of the numbers is 4.

    (c) The unique mode of the numbers is 3.

    Given exactly two of the statements are true, what is the maximum possible value of any of the numbers?

31. Let $x!$ be the function that finds the product of all positive integers less than or equal to x. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the remainder when the sum $(1!)^2 + (2!)^2 + (3!)^2 + ... + (100!)^2$ is divided by 100.

32. How many triangles with positive area and integer side lengths have a side of length 9 and a side of length 7?

33. Let $ABCD$ be a concave quadrilateral such that A is in the interior of $\triangle BCD$ and A lies on the segment connecting the midpoint M of BC and the midpoint N of CD. Given that $[BMND] = 21$, find $[ABCD]$.

34. Daniel and Tony each have a shape. Tony's shape has 14 more sides than Daniel's shape, and the sum of the interior angles of Tony's shape is triple the sum of the interior angles of Daniel's shape. How many sides does Tony's shape have?

35. Jeremy flips a coin, except it is not a regular coin, and has a probability $0 < h < 1$ of being heads. When Jeremy flips the coin 5 times, the probability of flipping 3 heads and 2 tails in some order is equal to the probability of flipping 5 heads in a row. If h can be written as $\frac{a - \sqrt{b}}{c}$ where a, b, and c are positive integers, $\gcd(a, c) = 1$, and b isn't divisible by any perfect squares greater than 1, find $a + b + c$.

36. How many three-digit numbers are there that use only the digits 1, 2, 3, 4, and 5 (each can be used more than once) and are divisible by 3?

37. Devin draws square $XMYC$ with side length 4. He picks a random point A inside of the square. If the probability that the area of triangle CAM is at least 4 can be expressed as $\frac{p}{q}$ in simplest form, find $p + q$.

38. Mythreya says that a number x is $N\%$ prime if 1 and x make up $N\%$ of its factors. For example, 6 would be 50% prime, and 16 would be 40% prime. By this (made-up) definition, what is the smallest number which would be 10% prime?

39. Find the unique positive integer m such that there exists exactly 551 integers n such that

$$(9m+n)x^2 - (m+n)x + m$$

has no real roots.

40. There exists a quadrilateral $ABCD$ such that $\angle B = 77^\circ$, $\angle D = 103^\circ$, and $\angle A = \angle C$. The two diagonals of this quadrilateral intersect at point E. Given that $AD = 10$, $AB = 24$, and the distance from the midpoint of $\overline{BD}$ to point E is 5, find the value of $AE \cdot CE$.