Joe Holbrook Memorial Math Competition

7th Grade

October 20, 2024

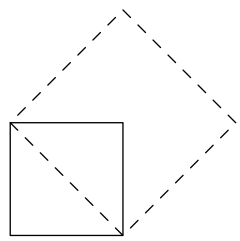
General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What number ♠ satisfies the equation ♠ + ♠ + ♠ = 27?
2. Bob has 10 bananas. If he gives 3 bananas to each of his friends and has 1 left, how many friends does he have?
3. What number is halfway between 11 and 111 on the number line?
4. Compute $(2 + 0 + 2 + 4) \times (20 + 24)$.
5. Brian is taking his JHMMC test, which is 75 minutes. He falls asleep for 20% of the test but stays awake for the rest. How many minutes was Brian awake?
6. What is the value of 20.24×5, rounded to the nearest integer?
7. Gabriele starts drawing squares, each with an area of 4. None of the squares overlap or share an edge with each other. Once the total area of all of Gabriele's squares added together is 100, he stops. What is the total perimeter of all of Gabriele's squares?
8. One purlunkle equals 10 inches. How many purlunkles equals 20 feet? (Note that there are 12 inches in a foot.)
9. Annabelle and Anastasia are 16 years apart, and two years ago Annabelle's age was 3 times Anastasia's age. In how many years will Annabelle be 2 times Anastasia's age?
10. Two squares are placed such that a diagonal of one square is a side of the other square. How many times bigger is the area of the larger (dashed) square than the smaller square?



11. Andy visits his grandmother today, a Saturday, and keeps doing so once every 17 days. How many days from now will he visit his grandmother on a Sunday?
12. A palindrome is a number which reads the same forwards and backwards. For example, both 22 and 494 are palindromes. Two positive integers x and $x + 13$ are two and three digit palindromes, respectively. Compute x.
13. Sabrina and Amy are crossing the street to the opposite corner of an intersection. Sabrina crosses one crosswalk before crossing the perpendicular crosswalk, while Amy cuts across the intersection diagonally. If they both start and end in the same places, one crosswalk is 16 feet, and the perpendicular crosswalk is 12 feet, how many more feet does Sabrina have to walk than Amy?
14. Alex picks socks out of his sock drawer every morning, one at a time. However, Alex does not know how many pairs of socks he owns! This morning, he wants to pick 8 individual socks from his drawer. If Alex wants to guarantee that he picks out at least one pair of socks, what is the greatest number of distinct pairs of socks that could be in the drawer?
15. Alice wants to collect some nonzero number of hexagons and pentagons. She wants the total number of sides of the hexagons to equal the total number of sides of the pentagons. What is the minimum number of shapes she needs in total?
16. Increasing the length of a rectangle by 25 percent and the width by x percent doubles the rectangle's area. Find x.
17. What is the remainder when $41^2 - 4$ is divided by 7?
18. My candle has n different colors initially, each in equal quantity, one on top of another. After burning it 70% of the way, only 3 colors are still visible in some amount. Find the sum of all possible initial values of n.

19. It takes Jeremy 30 minutes to walk from his house to Andrea's house. If he walks half the distance to Andrea's house, and bikes the rest, it takes him 20 minutes total. If Jeremy's walking speed is two miles per hour, in miles per hour, how fast does he bike?

20. What is the largest prime divisor of $2^{2024} - 2^{2022}$?

21. When a number is converted to base 8, the product of the digits in its base 8 representation is 40 (in base 10). What is the minimum value of this number, written in base 10?

22. Twenty-six people sit in a circle. At random, each person takes a marble from a jar with 24 indistinguishable green marbles, 1 blue marble, and 1 red marble. The person with the blue marble and the person with the red marble are not beside each other. How many distinct ways are there to distribute the marbles in this way?

23. Daniel and Tony each have a shape. Tony's shape has 14 more sides than Daniel's shape, and the sum of the interior angles of Tony's shape is triple the sum of the interior angles of Daniel's shape. How many sides does Tony's shape have?

24. From a deck of 10 cards labeled with the numbers 1 through 10, 5 cards are chosen in order without replacement. The probability that the cards are chosen in decreasing order can be represented by the fraction $\frac{a}{b}$ in simplest form. Compute $a + b$.

25. Alice has 3 positive integers a, b, and c. She tells you 3 statements:

    (a) The mean of the numbers is 3.
    (b) The median of the numbers is 4.
    (c) The unique mode of the numbers is 3.

    Given exactly two of the statements are true, what is the maximum possible value of any of the numbers?

26. Shirley wants to build a giant ice cream cone with a radius of r meters and height of 100 meters, as well as a giant sphere of ice cream with a radius of r meters. If she wants the cone and sphere to have the same volume, what must r be?

27. Andrea has a box of 10 marbles. Of them, 5 are red, 3 are green, and 2 are blue, and she randomly picks 3 marbles without replacement. If the probability that all 3 marbles are different colors is $\frac{m}{n}$ in simplest form, what is $m + n$?

28. Find the smallest 6-digit number $\overline{ABCDEF}$ (digits not necessarily distinct) such that $2 \mid \overline{AB}$, $3 \mid \overline{ABC}$, $4 \mid \overline{ABCD}$, $5 \mid \overline{ABCDE}$, $6 \mid \overline{ABCDEF}$. (The notation $n \mid x$ says that x is divisible by n.)

29. Jacob has two fair 6-sided die. If the probability the sum of numbers he rolls is a perfect square is $\frac{p}{q}$ in simplest form, what is $p + q$?

30. Daniel is very good at juggling oranges, and decides to juggle for 10 hours straight. At the beginning of the first hour, he is juggling 3 oranges. At the end of each hour, he has a $\frac{1}{2}$ chance of adding 1 orange and a $\frac{1}{3}$ chance of adding 2 oranges. If the expected number of oranges he is juggling at the end of the 10th hour can be expressed as $\frac{a}{b}$ in simplest form, what is $a + b$?

31. Let $x!$ be the function that finds the product of all positive integers less than or equal to x. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the remainder when the sum $(1!)^2 + (2!)^2 + (3!)^2 + ... + (100!)^2$ is divided by 100.

32. Alice wants to fill in a grid with the numbers $2, 3, \ldots, 10$ such that the numbers in any two adjacent squares (i.e. squares sharing an edge) are relatively prime, meaning they share no common factors greater than 1. Bob has already partially filled in the grid as shown below. How many ways are there to place the remaining numbers to satisfy Alice's condition?

10		
9	4	

33. Cyclic pentagon $ABCDE$ satisfies $AB = 5$, $CD = 5$, $DE = 6$ and $AE = 8$ with AD being a diameter of the circumscribed circle of $ABCDE$. Find the length of BC.

34. Jeremy flips a coin, except it is not a regular coin, and has a probability $0 < h < 1$ of being heads. When Jeremy flips the coin 5 times, the probability of flipping 3 heads and 2 tails in some order is equal to the probability of flipping 5 heads in a row. If h can be written as $\frac{a-\sqrt{b}}{c}$ where a, b, and c are positive integers, $\gcd(a, c) = 1$, and b isn't divisible by any perfect squares greater than 1, find $a + b + c$.

35. How many three-digit numbers are there that use only the digits 1, 2, 3, 4, and 5 (each can be used more than once) and are divisible by 3?

36. How many non-empty subsets of $\{1, 2, 3, \ldots 7\}$ are there such that the product of the largest and smallest element is divisible by 4?

37. Mythreya says that a number x is $N\%$ prime if 1 and x make up $N\%$ of its factors. For example, 6 would be 50% prime, and 16 would be 40% prime. By this (made-up) definition, what is the smallest number which would be 10% prime?

38. Distinct prime numbers p, q, and r satisfy the equation that $pq + qr + pr + pqr = 617$. Find the value of $p + q + r$.

39. There exists a quadrilateral $ABCD$ such that $\angle B = 77^\circ$, $\angle D = 103^\circ$, and $\angle A = \angle C$. The two diagonals of this quadrilateral intersect at point E. Given that $AD = 10$, $AB = 24$, and the distance from the midpoint of $\overline{BD}$ to point E is 5, find the value of $AE \cdot CE$.

40. Find the number of nonnegative integers less than 3^7 that have two consecutive ones in their base 3 representation. (An example of such an integer is 1121_3, which equals 43 in base ten.)