Joe Holbrook Memorial Math Competition

8th Grade

October 20, 2024

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Jeremy is randomly selecting letters from the alphabet. After he selects a letter, he cannot select it again. How many letters must he pick to guarantee that he picks at least one vowel (a, e, i, o, u, or y)?

2. Compute $(2 + 0 + 2 + 4) \times (20 + 24)$.

3. Maxwell is looking across the a river towards New York City. He spots a skyscraper perfectly North-East, and then turns to see a park which is perfectly South-East. They are both $7\sqrt{2}$ away from Maxwell. How far apart is the skyscraper from the park?

4. Arnav is practicing archery! He has a 10% chance of hitting a bullseye on each shot, but his friend will only give him a cookie if he hits a bullseye on both of his next two shots! If the chance Arnav gets a cookie can be written as $\frac{m}{n}$ in simplest form, what is $m + n$?

5. If $a\#b = \frac{a+b}{b}$ then what is $6\#((9\#3)\#4)$?

6. Calculate $26^2 + 31^2 - 25^2 - 30^2$.

7. Andy visits his grandmother today, a Saturday, and keeps doing so once every 17 days. How many days from now will he visit his grandmother on a Sunday?

8. David drives at 80 mph for an hour, and then 90 mph until he reaches his destination. His average speed throughout the drive was 84 mph. How many minutes was the whole drive?

9. Cristiano is trying to qualify for a soccer tournament. Currently, he has won 40% of the 20 games he has played. If Cristiano needs to win at least 75% of his games to qualify for the soccer tournament, how many more games does he need to play to qualify, assuming he wins all of the games?

10. What is the largest prime divisor of $2^{2024} - 2^{2022}$?

11. Sabrina and Amy are crossing the street to the opposite corner of an intersection. Sabrina crosses one crosswalk before crossing the perpendicular crosswalk, while Amy cuts across the intersection diagonally. If they both start and end in the same places, one crosswalk is 16 feet, and the perpendicular crosswalk is 12 feet, how many more feet does Sabrina have to walk than Amy?

12. Joey has studies 3 subjects - mathematics, physics, and English - and owns 4, 2, and 3 textbooks identical to each other given that they are the same subjects, respectively. He wants to line these 9 books up on a shelf such that the physics textbooks stay together. How many ways can Joey line up his books?

13. Help! Thor is trying to guess my secret number. Given that it is between 70 and 79, all I can tell Thor is that my number is divisible by the number of factors it has. He is able to guess my number after that clue. What is it?

14. Alice wants to collect some nonzero number of hexagons and pentagons. She wants the total number of sides of the hexagons to equal the total number of sides of the pentagons. What is the minimum number of shapes she needs in total?

15. Increasing the length of a rectangle by 25 percent and the width by x percent doubles the rectangle's area. Find x.

16. When a number is converted to base 8, the product of the digits in its base 8 representation is 40 (in base 10). What is the minimum value of this number, written in base 10?

17. What are the last two digits of 2024^{2024}?

18. Alex has a balloon with a volume of 40 cubic inches and a surface area of 32 square inches. When the balloon expands to 320 cubic inches, Alex observes that it keeps the same shape. What is the new surface area of the balloon?

19. Andrea has a box of 10 marbles. Of them, 5 are red, 3 are green, and 2 are blue, and she randomly picks 3 marbles without replacement. If the probability that all 3 marbles are different colors is $\frac{m}{n}$ in simplest form, what is $m + n$?

20. Daniel and Tony each have a shape. Tony's shape has 14 more sides than Daniel's shape, and the sum of the interior angles of Tony's shape is triple the sum of the interior angles of Daniel's shape. How many sides does Tony's shape have?

21. Magikarp and Bulbasaur decide to run in the same direction of a circular track that has a length of 400 meters. Magikarp can run at a constant rate of 40 meters per hour and Bulbasaur can run at a constant rate of 80 meters per hour. How much time in hours will it take for the two to meet each other 69 times excluding the time when they start together?

22. Chris has invested in some state of the art crayons for his art project. He has red, blue, and green crayons. He tells you the following information:

    (a) All but 7 of my crayons are red.

    (b) All but 3 of my crayons are blue.

    (c) All but 8 of my crayons are green.

    How many total crayons does he have?

23. Suppose we have $\triangle ABC$ such that $AB = 5$, $BC = 8$, and $\triangle DEF$ such that $DE = 15$, $EF = 24$, and $\angle ABC$ has the same angle measure as $\angle DEF$. If the area of $\triangle DEF$ is 18, find the area of $\triangle ABC$.

24. Find the sum of the largest and smallest base 10 numbers such that both their base 5 and base 7 representations have 5 digits.

25. Given that a and b are positive integers that satisfy $6ab - 3a + 2b = 29$, find the sum of the possible values of ab.

26. Bessie the cow is standing at $(0, 0)$ on the coordinate plane, and Farmer John is standing at $(6, 6)$. Every step, Bessie can move to an adjacent lattice point (1 unit north, south, east, or west). However, a sinkhole has appeared underneath $(5, 5)$, $(5, 4)$, and $(4, 5)$, meaning Bessie cannot move to any of those points. How many ways does Bessie have to reach Farmer John in a total of 12 steps?

27. Let $x!$ be the function that finds the product of all positive integers less than or equal to x. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the remainder when the sum $(1!)^2 + (2!)^2 + (3!)^2 + ... + (100!)^2$ is divided by 100.

28. Jacob has two fair 6-sided die. If the probability the sum of numbers he rolls is a perfect square is $\frac{p}{q}$ in simplest form, what is $p + q$?

29. Alice wants to fill in a grid with the numbers $2, 3, \dots, 10$ such that the numbers in any two adjacent squares (i.e. squares sharing an edge) are relatively prime, meaning they share no common factors greater than 1. Bob has already partially filled in the grid as shown below. How many ways are there to place the remaining numbers to satisfy Alice's condition?

10		
9	4	

30. Let ω be a circle centered at O. Let P be an arbitrary point on the circumference of ω. The line l is drawn such that it is tangent to ω at P. Finally, the angle bisector of the two angles formed by $\overline{QR}$ and l intersects ω at Q and R. If $\overline{QR} = 2024$. Find the radius of ω.

31. Jeremy flips a coin, except it is not a regular coin, and has a probability $0 < h < 1$ of being heads. When Jeremy flips the coin 5 times, the probability of flipping 3 heads and 2 tails in some order is equal to the probability of flipping 5 heads in a row. If h can be written as $\frac{a - \sqrt{b}}{c}$ where a, b, and c are positive integers, $\gcd(a, c) = 1$, and b isn't divisible by any perfect squares greater than 1, find $a + b + c$.

32. How many three-digit numbers are there that use only the digits 1, 2, 3, 4, and 5 (each can be used more than once) and are divisible by 3?

33. Mythreya says that a number x is $N\%$ prime if 1 and x make up $N\%$ of its factors. For example, 6 would be 50% prime, and 16 would be 40% prime. By this (made-up) definition, what is the smallest number which would be 10% prime?

34. Let ABC be a triangle with $AB = 8$, $BC = 15$, and AC be the diameter of the circumcircle of ABC. A ray from B intersects AC at D and bisects the arc AC. If $AD = \frac{m}{n}$ in simplest form, then find $m + n$.

35. How many non-empty subsets of $\{1, 2, 3, \ldots 7\}$ are there such that the product of the largest and smallest element is divisible by 4?

36. Distinct prime numbers p, q, and r satisfy the equation that $pq + qr + pr + pqr = 617$. Find the value of $p + q + r$.

37. Point P is on side AB of rectangle $ABCD$ such that $\frac{AP}{PB} = \frac{5}{6}$. Line segments DP and CP are drawn, cutting the rectangle into three triangular pieces. The ratio between the area of the largest piece and the area of the smallest piece can be written as $\frac{p}{q}$ in simplest form. Find $p + q$.

38. Bob went to the store and bought two items. The first item cost a positive integer number of dollars, while the second item didn't cost a positive integer number of dollars but did cost a positive integer number of cents. However, the cashier accidentally charged him the product of the two prices, which he noticed was the same as the actual charge he should have received. How many possibilities are there for the prices of the two items?

39. In triangle ABC, let D be the foot of the altitude from A to line BC, and let E be the midpoint of BC. Triangle ABC has the property that its area is equal to the product of the lengths of AD and AE. Given that $AB = 5$ and $AC = 6$, the quantity AD^2 can be written as $\frac{p}{q}$ in simplest terms. Find $p + q$.

40. The following sum has value S such that it can be represented as $\frac{p}{q}$ where p and q are relatively prime:

$$\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\sum_{c=0}^{\infty}\frac{a+b+c}{3^{a+b+c}}$$

Compute $p + q$.