Joe Holbrook Memorial Math Competition

4th Grade

October 20, 2024

1. $2 \cdot 2 + 0 + 4 \cdot 4 \cdot 4 + 2 \cdot 2 = 4 + 64 + 4 = \boxed{72}$.

2. Observe that $\spadesuit + \spadesuit + \spadesuit = 3 \times \spadesuit = 27$, which implies $\spadesuit = 27 \div 3 = \boxed{9}$.

3. We want the average of 11 and 111, which is $(11 + 111) \div 2 = \boxed{61}$.

4. $(2 + 0 + 2 + 4) \times (20 + 24) = 8 \times 44 = \boxed{352}$.

5. If Brian was asleep for 20% of the test, it means he must have been awake for 80% of the test. Therefore, Brian was awake for $75 \times 0.8 = \boxed{60}$ minutes.

6. The least positive multiple of 11 is 11, and the greatest positive multiple of 11 less than 200 is 198 (which is equal to 11×18). Thus, the total amount of multiples satisfying this question is $\boxed{18}$.

7. $50 * 180 = 9000$ minutes, which is equal to $\boxed{150}$ hours.

8. Since 50 students completely fill 2 buses, a single bus must have a maximum capacity of $50 \div 2 = 25$ students. So 289 students would need at least $289 \div 25 = 11.56$ buses, or $\boxed{12}$ buses.

9. If Tony the Tree was 26 feet tall yesterday, it means Tony the Tree will be $26 \times 2 = 52$ feet tall today, and tomorrow he will be $52 \times 2 = \boxed{104}$ feet tall.

10. He buys $10 \times 90 = 900$ sticky notes in total. He uses $3 \times 180 = 540$ of them. Therefore, he has $900 - 540 = \boxed{360}$ remaining.

11. We know 20 feet equals 20×12 inches. So, $\dfrac{20 \times 12}{10}$ is the number of purlunkles you need, or $\boxed{24}$.

12. We have
$$2024 = 2025 - 1 = 45^2 - 1 = (45 - 1)(45 + 1) = 44 \cdot 46 = 2^3 \cdot 11 \cdot 23.$$
The desired sum is
$$2 + 11 + 23 = \boxed{36}.$$

13. Arnav has a $\dfrac{1}{10}$ probability of hitting a bullseye on each shot. Since the shots are independent, the probabilities multiply. The probability of getting two bullseyes in a row is $\dfrac{1}{10} \times \dfrac{1}{10} = \dfrac{1}{100}$, or $\boxed{101}$ as desired.

14. Using the difference of squares factorization,
$$41^2 - 4 = (41 - 2)(41 + 2) = 39 \cdot 43.$$
39 has remainder 4 when divided by 7, and 43 has remainder 1, so their product has remainder $\boxed{4}$.

15. Increasing the length multiplies the area by $\frac{5}{4}$, so we must multiply the width by $\frac{8}{5}$ to double the area, since $\frac{5}{4} \cdot \frac{8}{5} = 2$. $\frac{8}{5}$ equals 160 percent, so the answer is $\boxed{60}$.

16. The only two digit palindromes that becomes a three digit number when 13 is added to it are 88 or 99, and testing shows that only $x = \boxed{88}$ works.

17. If Caleb has a dollar of pennies and a dollar of dimes, he has exactly 100 pennies and 10 dimes, which adds to 110 coins. If he loses half of these, he will lose 55 coins. Assuming he lost the least amount of money possible, he will only lose pennies, meaning he loses 55 cents and keeps $\boxed{145}$.

18. Consider the worst case, in which he picks distinct pairs of socks right until his last pick. This would mean that for 7 picks in a row he needs distinct socks, and therefore $\boxed{7}$ pairs of distinct socks. Notice that on his 8th pick it does not matter what sock he picks, as whatever he picks it will be a pair with another one already chosen. This situation can also be modeled with 7 pigeonholes and 8 pigeons.

19. Let a be an angle of the triangle. The other 2 angles can be expressed as $3a$ and $9a$. Since we know that the sum of the angles of a triangle is 180 degrees, we can write the equation $a + 3a + 9a = 13a = 180$, $a = \frac{180}{13}$. The closest integer to this value is $\boxed{13}$.

20. Let the side length of one square be x. Then, the square will have diagonal length $x\sqrt{2}$, and since $x\sqrt{2} > x$, the smaller square will have side length x and the larger square with have side length $x\sqrt{2}$. Hence, the desired answer is
$$\frac{(x\sqrt{2})^2}{x^2} = \frac{2x^2}{x^2} = \boxed{2}.$$

21. Since the median is 20, this must be the average of the 4th and 5th largest numbers in the list. Suppose they are both 20. Since we want the smallest minimum possible, assume the 6th, 7th, 8th numbers are all 34. Then, we can also make the 2nd and 3rd numbers both 20. So, our list is $20, 20, 20, 20, 34, 34, 34$, leaving out the smallest number. Since the mean is 17, the total sum would be $8 \cdot 17 = 136$. The sum of the 7 numbers we have is 182, so the remaining number must be $136 - 182 = -46$, so the answer is $\boxed{46}$.

22. It takes Jeremy 15 minutes to walk half the way to Andrea's house. Therefore, it took Jeremy 5 minutes to bike the same distance. So, Jeremy bikes $15 \div 5 = 3$ times faster than he walks. So, his biking speed is $\boxed{6}$ miles an hour.

23. $4\Delta 5 = 4 \times (4 + 5) = 36$

    $3\Delta 36 = 3 \times (3 + 36) = \boxed{117}$

24. We can break this problem into cases based on how many houses Dolly builds. If she builds 1 house, there are 6 different spots where she could build it. If she builds 2 houses, she can start by placing 1 house on one of the 6 vertices, and since she can't place a house on either of the adjacent vertices, she has 3 choices for the second house. $6 \times 3 = 18$ choices for placing these houses, but each choice is counted twice, so there are actually $\frac{18}{2} = 9$ ways of placing 2 houses. If there are 3 houses, Dolly must places them on alternating vertices, giving 2 ways of placing 3 houses, and it is apparent that trying to place more than 3 houses would result in multiple houses being adjacent, which Dolly does not want. Altogether, this gives $6 + 9 + 2 = \boxed{17}$ ways for Dolly to build new houses.

25. The coins used would be 1 quarter, 2 dimes, and 4 pennies, for $\boxed{7}$ coins.

26. Every visit, the day of the week advances by 3 days, and we need to move 1 day ahead of Saturday. This occurs after 5 visits, because $5 \cdot 3 = 15$ leaves a remainder of 1 when divided by 7. Thus, after $\boxed{85}$ days, the day will be Sunday.

27. If 6 mome = 1 tove, then 18 mome = 3 tove, which equal 14 borogoves. Since 2 borogoves = 5 jabberwocks, 14 borogoves = 35 jabberwocks. Then, 18 mome = 35 jabberwocks, so $a = 18$, $b = 35$ and $a + b = \boxed{53}$.

28. Out of the original 20 games, Cristiano has won $20 \times 0.4 = 8$ of the games. We can write the fraction $\frac{8}{20}$, with the numerator being the amount of games Cristiano has won and the denominator being the total amount of games Cristiano has played. Let x be the amount of extra games Cristiano plays. Since Cristiano needs to win at least 75% of his total games, we can write the expression $\frac{8+x}{20+x} = 0.75 = \frac{0.75}{1}$. We can cross multiply to get $8 + x = 15 + 0.75x$. Simplify to get $0.25x = 7$, and then $x = \boxed{28}$.

29. Notice that each of the 6 pairs of numbers that are formed by taking 2 distinct values from $1, 2, 3, 4$ are either two adjacent numbers in a row, two adjacent numbers in a column, or diagonally opposite numbers. Therefore, we have that the sum is equal to
$$1 \cdot 2 + 1 \cdot 3 + 1 \cdot 4 + 2 \cdot 3 + 2 \cdot 4 + 3 \cdot 4 = 35$$
and subtracting out the sum of the product of the values on each diagonal.

    The sum of the product of the values on each diagonal is either $1 \cdot 2 + 3 \cdot 4 = 14$, $1 \cdot 3 + 2 \cdot 4 = 11$ or $1 \cdot 4 + 2 \cdot 3 = 10$. To maximize the sum of the product of the values in each row and each column, we need to minimize the sum of the product of the values each diagonal. Therefore, our desired answer is $35 - 10 = \boxed{25}$.

30. Let h and p be the number of hexagons and pentagons, respectively. We need $6h = 5p$, which means we need to find the least common multiple of 6 and 5. This is 30, so we want 5 hexagons and 6 pentagons to satisfy this equation with the least number of shapes. Our answer is thus $5 + 6 = \boxed{11}$.

31. Consider the worst case scenario: everytime a new person comes to watch the game, they have a different initial. There are a total of $26^2 = 676$ possible initials, so in the worst case there will be 676 people with different initials. To guarantee overlap, we simply need one more seat than 676, giving us a minimum of $676 + 1 = \boxed{677}$ seats.

32. Using the Pythagorean theorem, Amy must walk $\sqrt{12^2 + 16^2} = 20$ feet. Sabrina walks a total of $12+16 = 28$ feet, so the difference is $28 - 20 = \boxed{8}$.

33. Notice that the triangles could have been stacked on top of each other resulting in a shape with 3 sides, they could share exactly one side resulting in a shape with 3 or 4 sides, or they could share a fraction of one side resulting in a shape with 5 or 6 sides. So, we get the answer is $3 + 4 + 5 + 6 = \boxed{18}$.

34. Because $DE = 3AB$, $EF = 3BC$, and $\angle ABC \equiv \angle DEF$, by SAS similarity, $\triangle ABC \sim \triangle DEF$. Because the side lengths are at a ratio of $1 : 3$ for $\triangle ABC$ to $\triangle DEF$, the areas' ratio must be the square of $1 : 3$, i.e. $1 : 9$. Thus, $\frac{[ABC]}{[DEF]} = \frac{1}{9}$, and since $[DEF] = 18$, the area of $\triangle ABC$ is $\boxed{2}$.

35. Let P be closer to point B. Since it is a $30 - 60 - 90$ triangle, we know by similar triangles that P is the foot of the altitude to base BC. Then, by angle bisector theorem we know $\frac{CQ}{PQ} = \frac{2}{1}$. Since triangle QBC is isosceles, we know $PQ = \frac{60}{4} = \boxed{15}$.

36. Let r, b, and g be the number of red, blue, and green crayons, respectively. Then, we can set up the following equations:

    (a) $b + g = 7$
    (b) $r + g = 3$
    (c) $r + b = 8$

    Adding up all the equations, we get $2r + 2b + 2g = 18$. Therefore, the total number of crayons, $r + b + g$, equals $\boxed{9}$.

37. Note that at $x = 5$ and above, $x!$ will be a multiple of 10 and thus $(x!)^2$ will be a multiple of 100. Thus, the remainder when the infinite sum is divided by 100 is the same as the remainder when $(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2$ is divided by 100. $(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2 = 1^2 + 2^2 + 6^2 + 24^2 = 1 + 4 + 36 + 576 = 617$, which has a remainder of $\boxed{17}$ when divided by 100.

38. We can first see that B, D, and F must all be even and D must be either 0 or 5. Since we are trying to find the minimum number, we can first assume that $A = 1$ to try to find a minimal case. One thing that we also need to realize is that $3|A + B + C$ and $3|A + B + C + D + E + F$. Since we know $3|A + B + C$, $3|D + E + F$. In order to find a small value of $\overline{ABCDEF}$, we need to try to minimize the values in the higher digits. Since there is no other restriction for B except it must be even, we can also set $B = 0$. As $A = 1$ and $B = 0$, $C = 2, 5, 8$. Since C is the next biggest number we can control, we set C as 2 to minimize it. The last condition we have to keep is that $4|\overline{CD}$ so $D = 0, 4, 8$. Once we set $D = 0$, we can see that we can automatically set $E = F = 0$ and see that this satisfies all the conditions. Our answer will be $\boxed{102000}$

39. Parity argument gives that one of the primes, WLOG r, is 2, since 617 is odd and the sum of four odd numbers is even. Therefore, $pq + 2q + 2p + 2pq = 617$, so $3pq + 2q + 2p = 617$, so $9pq + 6q + 6p = 1851$, and $9pq + 6q + 6p + 4 = 1855$. Thus $(3p + 2)(3q + 2) = 1855 = 5 \cdot 7 \cdot 53$, so the only possibility is p and q are 17 and 11. Our answer is $2 + 11 + 17 = \boxed{30}$.

40. Notice that such a value of x must satisfy $\gcd(x, x+72) = \gcd(x, 72) = 1$. We can then use complimentary counting.

    Notice that $72 = 2^3 \cdot 3^2$. There are $\lfloor \frac{2024}{2} \rfloor = 1012$ multiples of 2 between 1 and 2024, and there are $\lfloor \frac{2024}{3} \rfloor = 674$ multiples of 3 between 1 and 2023. Of these, $\lfloor \frac{2024}{6} \rfloor = 337$ are both multiples of 2 and 3 and are counted twice, meaning there are a total of $1012 + 674 - 337 = 1349$ values of x between $1 \leq x \leq 2024$ that are divisible by 2 or 3, meaning that there are $2024 - 1349 = \boxed{675}$ integers that are not divisible by 2 or 3, meaning that they are relatively prime to 72.